LEARNING MATERIAL OF

FLUID MECHANICS

PREPARED BY – ER. KAMALA KANTA TRIPATHY

&

ER. BIKASH RANJAN SAHU

Ref: 1) Cengel
2) Frank M. White

problems practice → Subrahmanyan
Theory → Modi & Seth

Easy start → R.K. Bansal, K.L. Kumar
Jadgish Lal -- etc.

Volten Lewis
MIT - physics

Fluid Mechanics [6 - 10 Marks]

Def^n

→ **Fluid** : fluid is a substance which deform continuously for a small amount of shear force also.

Solid → S.M.

Incompress fluid → Liq^n } fluid (open channel flow)
Compress fluid → Gas

→ **Introduction - [properties]**

1) Density, mass density, specific mass 'ρ'.

$\rho = \frac{mass}{Vol^m} = \frac{kg}{m^3} = ML^{-3}$

$\rho_{water} = 1000\ kg/m^3$, $\rho_{air} = 1.2\ kg/m^3$

$\rho_{sea\ water} = 1025\ kg/m^3$

$\rho_{Ice} = 915\ kg/m^3$

→ ρ_{water} is Maximum at $4°C$

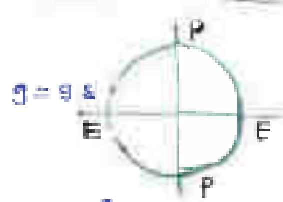
Moist air

Dry air   Water vapour

2) Specific weight (γ) = Weight density

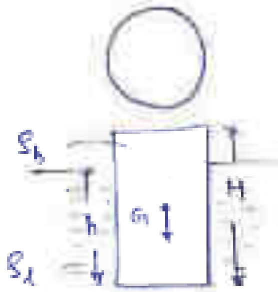
$\gamma = \frac{Weight}{Vol^m} = \frac{N}{m^3}$ $\boxed{W = mg}$ $\boxed{\gamma = \rho g}$

$\gamma_w = \rho_w \times g$
$= 1000 \times 9.81$
$\boxed{\gamma_w = 9810\ N/m^3}$



$g_P > g_E$

3) Weight — $\boxed{W = mg = F}$



$W_b = \gamma_b \times Vol_b$

$= \rho_b \cdot g \times \frac{\pi}{4} d_b^2 H$

$W_{liq} = \gamma_{liq} \times Vol_{liq}$

$= \rho_{liq} \cdot g \times \frac{\pi}{4} d_{liq}^2 h$

→ Bulk modulus $K = \frac{\text{Direct stress}}{\text{Volumetric strain}} = \frac{dp}{(dV/V)} = \frac{dp}{[d\rho/\rho]}$

4) Viscosity

→ It is a property of a fluid by virtue of which, it offers resistance for the movement of one layer over the other.

→ It due to 1) cohesion → liq^n
2) Molecular Momentum exchange → gas

Note → Molecular Momentum Exchange (↑) → viscosity (↓)

Effect of Temp:

① Liquid → T ↑, μ ↓

$\boxed{\mu \propto \frac{1}{T}}$

temp increase which break the bond of cohesion fluid so, it decrease

$$\mu_t = \frac{\mu_0}{1+\alpha T+\beta T^2}$$

② Gases → T ↑, μ ↑

High Randomness

$\boxed{\mu \propto T}$

$$\mu_t = \mu_0 + \alpha T - \beta T^2 \quad (\alpha >> \beta)$$

Effect of pressure :

→ With increase in pressure the viscosity increase for both liquid & gases but effect on the liquid is negligible.

Eg liq^n — P ↑ → 1 atm ⇒ 1000 atm
μ ↑ → 1 unit ⇒ 2 unit

→ Units of viscosity :

Dynamics μ (NS/m² or kg/ms)		Kinematics $\nu = \mu/\rho$ (m²/sec)	
SI	CGS	SI	CGS
$\frac{N.S}{m^2}$ or kg/m·sec	$\frac{Dyne\ sec}{cm^2}$ [poise]	$\frac{m^2}{sec}$	cm²/sec [stoke]

$1 \text{ poise} = 10^{-1} \frac{N.S}{m^2}$

$1 \text{ stoke} = 10^{-4} \text{ m}^2/s$

$1 N = 10^5 \text{ Dyne}$

→ Water :

$\rho = 1000$ kg/m³

$S = 1.0$

$K = 2.15$ GPa

$\mu = 1.02$ centipoise $= 1.02 \times 10^{-2}$ poise
$= 1.02 \times 10^{-2} \times 10^{-1}$ NS/m²

$\sigma = 0.073$ N/m

vapour pressure = 2.36 m of water [Abs]

→ Air

$\rho = 1.2$ kg/m³

$\mu = 1.47 \times 10^{-5}$ NS/m²

ISRO-10: For a given mass fluid when the pressure increases from 3 MPa to 3.5 MPa causing the density to increase from 500 kg/m³ to 501 kg/m³ then the Bulk modulus of fluid (K)

$$\rightarrow K = \frac{dP}{(dV/V)} = \frac{dP}{(d\rho/\rho)} = \frac{3.5 - 3.0}{1/500} = (0.5)(500)$$

$$\boxed{K = 250\ MPa}$$

DRDO-09: The increase in pressure required to decrease unit volume of mercury [K_{Hg} = 26.5 MPa] by 0.1 % is (?)

$$\cancel{K_{Hg} = \frac{dP}{(dV/V)} = \frac{0.99}{-dV/V} = 26.5}$$

$$K_{Hg} = \frac{+dP}{\left[-\frac{dV}{V}\right]} \Rightarrow +dP = K\left[-\frac{dV}{V}\right] = 26.5\times10^{6} \times \left[\frac{0.1}{100}\right]$$

$$\boxed{+dP = 26.5\ kPa}$$

ES: A liquid which specific gravity of 0.8 & dynamic viscosity is 10 poise, has kinematic viscosity = (?) (stoke)

$$\rightarrow \nu = \mu/\rho = \frac{10\times10^{-1}\ Ns/m^2}{0.8\times1000} = 12.5\times10^{-4}\ m^2/s$$

$$\boxed{\nu = 12.5\ stoke}$$

2) Soap bubble

$F_p = F_\sigma$

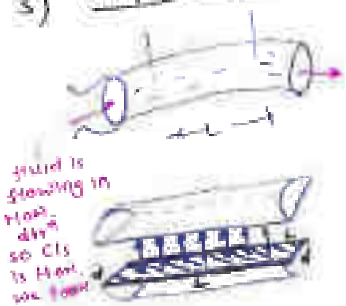
$p \times A_c = \sigma \times L$

$p \times \frac{\pi d^2}{4} = \sigma [\pi d_o + \pi d_i]$ — both externally & internally surface

but $d_o = d_i = d$

$$\boxed{p = \frac{8\sigma}{d}}$$

3) Liquid jet



$F_p = F_\sigma$

$p \times A_c = \sigma \times L$

$p \times L \times d = \sigma \times [2L + 2d]$ — but fluid flow along dirⁿ L not in dirⁿ of d so d is neglect

$$\boxed{p = \frac{2\sigma}{d}}$$

NOTE: Surface tension can also expressed as

$$\sigma = \frac{\text{Surface Energy}}{\text{Surface Area}} = \frac{\text{Joule}}{m^2} = \frac{N \cdot m}{m^2} = \frac{N}{m}$$

Work reqⁿ to burst a water droplet $\} = \sigma \times (4\pi R^2)$ $\qquad \frac{N}{m} \cdot m^2 = N \cdot m$

Ex One bubble is convert into 64 small bubble

Here is work reqⁿ to break bubble (Big) (work on system (-ve)

$E_1 + W = E_2$ $\Rightarrow$ $W = E_2 - E_1$ $\quad |\ E_1 = E_2 - W$

$\rightarrow E_1 = \sigma \times 4\pi R^2$

$\rightarrow E_2 = n\sigma \times 4\pi r^2$

but $V_1 = V_2 \Rightarrow \frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3 \Rightarrow r = R/n^{1/3}$

$W = n\sigma 4\pi r^2 - \sigma 4\pi R^2 = n\sigma 4\pi \left(\frac{R}{n^{1/3}}\right)^2 - \sigma 4\pi R^2$

$\boxed{W = 4\pi R^2 \sigma [n^{1/3} - 1]}$

→ Surface Tension :

	P
1) Water droplet	$\frac{4\sigma}{d}$
2) Soap bubble	$\frac{8\sigma}{d}$
3) Liquid jet	$\frac{2\sigma}{d}$

→ It is property of liquid surface film to exert tension, it is a force required to maintain unit length in equilibrium

Surface film

GATE

→ Surface tension because of cohesion it is varies inversely with temp

$$\boxed{\sigma \propto \frac{1}{T}}$$

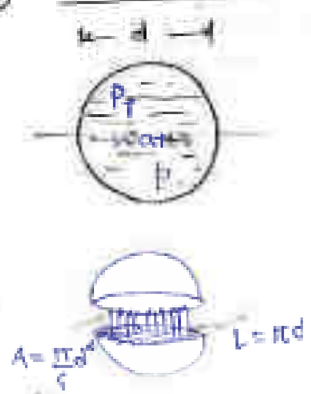
Ref^n → $\sigma_{water} = 0.073$ N/m @ 30 °C

$\sigma_{water} = 0.0589$ N/m @ 100 °C

$\sigma_{Hg} = 0.49$ N/m @ 30 °C

p = The pressure inside extremity in excess to outside Atm. pressure

1) Water droplet



$$\boxed{P_T > P_{atm}}$$

$$p = P_T - P_{atm}$$

$$F_{bursting} = F_{resist}$$

$$F_{pressure} = F_{\sigma}$$

$$p \times A = \sigma \times L$$

$$p \times \frac{\pi}{4} d^2 = \sigma \times \pi d \Rightarrow \boxed{p = \frac{4\sigma}{d}}$$

2) Soap bubble

$F_p = F_\sigma$

$P \times A_c = \sigma \times L$

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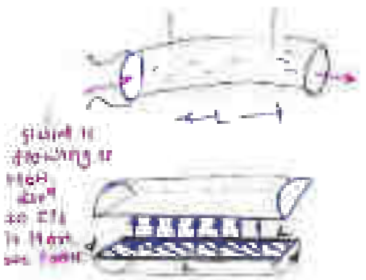

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$P \times L \times d = \sigma \times [2L + \cancel{2d}]$ — but liquid flow along dir L not in dirn of so d is ...

$$\boxed{P = \frac{2\sigma}{d}}$$

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Ex. One bubble is convert into 64 small bubble.

Here is work reqn to broke bubble (Big) (work on system (-ve))

$E_1 + W = E_2 \Rightarrow W = E_2 - E_1$ | $E_1 = E_2 - W$

→ $E_1 = \sigma \times 4\pi R^2$

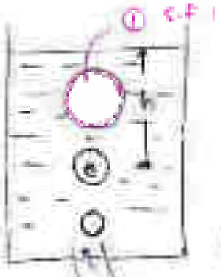
→ $E_2 = n\sigma \times 4\pi r^2$

but $V_1 = V_2 \Rightarrow \frac{4}{3}\pi R^3 = n\frac{4}{3}\pi r^3 \Rightarrow r = R/n^{1/3}$

$W = n\sigma 4\pi r^2 - \sigma 4\pi R^2 = n\sigma 4\pi \left(\frac{R}{n^{1/3}}\right)^2 - \sigma 4\pi R^2$

$$\boxed{W = 4\pi R^2 \sigma [n^{1/3} - 1]}$$

GATE-IS ES



Total pressure 'B' inside air bubble @ 'h'

(i) $\frac{4\sigma}{d}$ (ii) $\frac{8\sigma}{d}$

(iii) $\frac{4\sigma}{d} + \rho g h$ (iv) $\frac{8\sigma}{d} + \rho g h$

→ as air moving up, size increasing due to decrease in density of air is less

ES The maximum shear stress developed in lubricant oil having viscosity 0.981 poise, filled between two parallel plate & moving with velocity of 2 m/s is (?) which are 1 cm apart.

→ $\mu = 0.981 = 0.0981 \ Ns/m^2$

$y = 0.01$

$V = 2$

$\tau = \mu \frac{du}{dy} = 0.0981 \left(\frac{2}{0.01}\right)$

$\boxed{\tau = 19.62 \ N/m^2}$

ES For a flow over flat plate, the velocity profile given by $u = \frac{3}{4}y - y^2$, then shear stress at a location 0.30 metre above the plane is K times the shear stress at 0.2 m above the plane, then K = (?)

→ $u = \frac{3}{4}y - y^2 \Rightarrow \frac{du}{dy} = \frac{3}{4} - 2y$

$K = \frac{\tau_1}{\tau_2} = \frac{\mu \left.\frac{du}{dy}\right)_1}{\mu \left.\frac{du}{dy}\right)_2} = \frac{\frac{3}{4} - 2(0.3)}{\frac{3}{4} - 2(0.2)} = \frac{\frac{3}{4} - \frac{6}{10}}{\frac{3}{4} - \frac{4}{10}} = \frac{30-24}{30-16} = \frac{6}{14} = \boxed{3/7} = K$

Gate-04 A lubricating oil with specific gravity of 0.88 & kinematic viscosity of $\nu = 7.4 \times 10^{-7}\ m^2/s$ filled bet" two parallel plates if the top plate is moving with velocity of 0.5 m/s while the bottom one is stationary assume the linear velocity variation over a gap of 0.5 mm between the plates the max shear stress developed in Pa at the fixed plate (?)

$\rightarrow \nu = \frac{\mu}{\rho} \Rightarrow \mu = 7.4 \times 10^{-7} \times 0.88 = 6.51 \times 10^{-7} \times 1000$

$\mu = 6.51 \times 10^{-4}$

$\tau = \mu \frac{du}{dy} \Rightarrow \tau = 6.51 \times 10^{-4} \times \frac{0.5}{0.5 \times 10^{-3}} = 6.51 \times 10^{-1}$

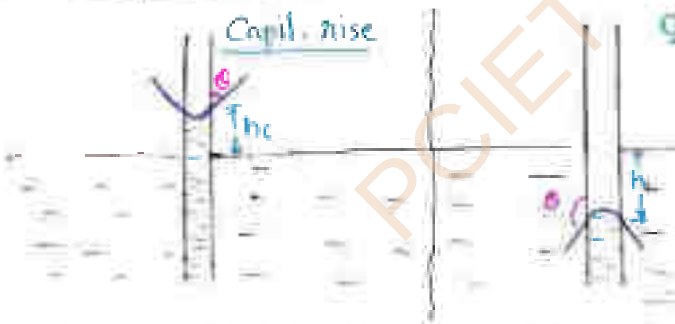
$\boxed{\tau = 0.651\ N/m^2} \leftarrow$ (max.)

Read :- velocity τ

→ parabolic ➡ linear
→ cubic ➡ parabolic
→ linear ➡ Const.

Thermometer works on ~~the~~ Zero^th law of Thermodynamics

⇒ Capillarity



→ d < 6 mm → capil c
→ d > 10 mm → Mano cho.

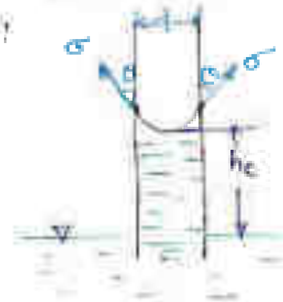
Cap. Rise	Cap. fall
→ Adh > coh	→ coh > Adh ✓
→ θ < 90°	→ θ > 90°
→ Concave	→ convex
Ex. water, kerosine	→ Ex. Hg

Adh > coh

coh > Adh

$$\boxed{h_c = \frac{4\sigma\cos\theta}{\gamma d}}$$

Proof :



$F_{\sigma}\uparrow = F_{w}\downarrow$ wt

$[\sigma\cos\theta]\,L = \gamma \times Vol^{m}$

$\sigma\cos\theta \times \pi d = \gamma \times \frac{\pi}{4}d^2 \times h_c$

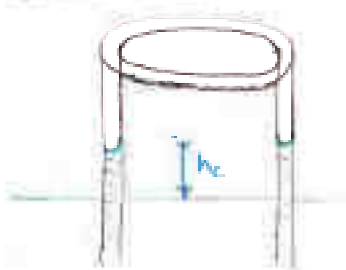
$$\boxed{h_c = \frac{4\sigma\cos\theta}{\gamma d}}$$

where → $\theta = 0$, pure water & glass

$= 25°$, contaminated water & glass

$= 128° \rightarrow$ Hg - glass

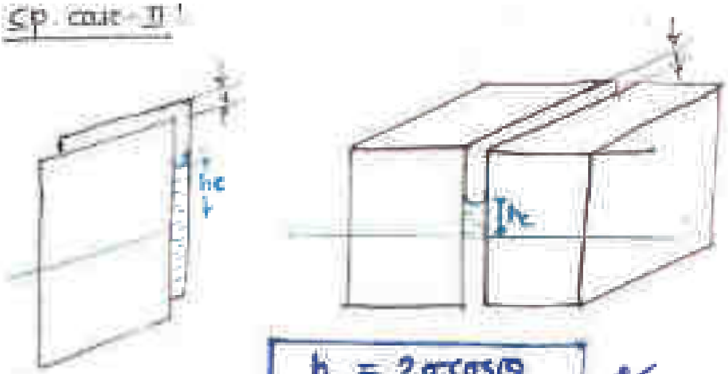
Sp. case-(i) :



$$\boxed{h_c = \frac{4\sigma\cos\theta}{\gamma(d_o - d_i)}}$$

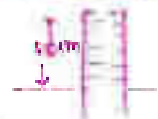
Where γ = specific wt.

Sp. case - II :

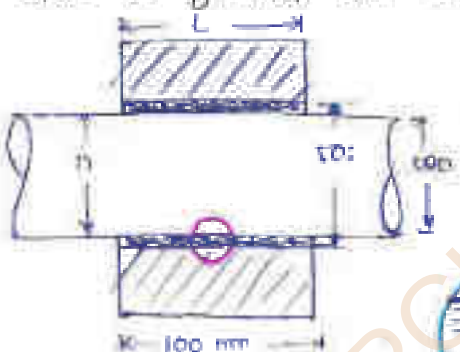


$$\boxed{h_c = \frac{2\sigma\cos\theta}{\gamma t}}$$

Note: If h_c = 55 cm & it height is 50 cm then that is over flow.



GATE '16 CIVIL: A circular shaft of 500 mm diameter is rotating inside a sleeve of 502 mm and 2000 rpm. The angular space was filled with lubricant of viscosity 5 poise. Calculate the power loss due to friction for sleeve length of 100 mm.



$\mu = 5 \times 10^{-1} \ \frac{Ns}{m^2}$

$D = 0.5$ m

$N = 2000$ rpm

$y = \frac{d_o - d_i}{2} = \frac{2}{2}$

$= 1$ mm

$V = \frac{\pi D N}{60}$

$\rightarrow P = \frac{2\pi N T}{60} \Rightarrow T = F \times dist$

$= F \times R$

$\tau = \frac{F'_{friction}}{A} = \mu \frac{du}{dy} \Rightarrow F' = (\pi d L)\, \mu \frac{du}{dy}$

$= \pi \times 0.5 \times 0.1 \times 5 \times 10^{-1} \times \frac{\left(\frac{\pi \times 0.5 \times 2000}{60}\right)}{0.001}$

$= 41.08$ N

$\rightarrow T = F' \times R = 41.08 \times \frac{0.5}{2} = 10.27$

$\rightarrow P = \frac{2\pi \times 2000 \times 10.27}{60} = 2.14$ kW

NOTE

$A = \pi d L$ $A = \frac{\pi}{4} d^2$ $A = 2\pi r^2$

(Surface area where part is moving or in rubbing action happen)

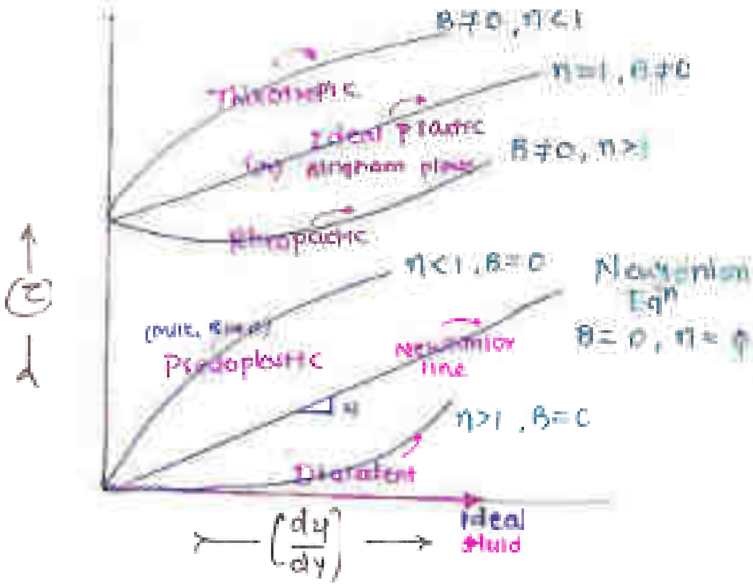
→ $P_{loss\ (friction)} = \left(\frac{\mu A V}{y}\right) \cdot V$ $\tau/y = w$

$\boxed{P_{loss} \propto \mu}$

Viscosity (↑) → power loss (↑)
(molecule)

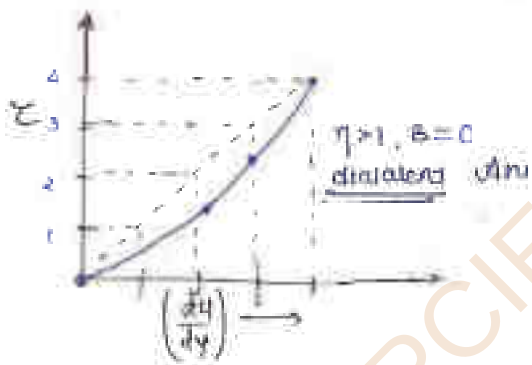
Classification of fluid

- Ideal
- Real
  - Newtonian
    - → $\tau = \mu \left(\frac{du}{dy}\right)$
    - → B.M
  - Non - Newtonian (IMP state PSU's)
    - → $\tau = A \left(\frac{du}{dy}\right)^n + B$
    - → Rheology
      - Time dependent
        - → Thixotropic
        - → Rheopectic
      - Time Independent
        - → Pseudoplastic
        - → Ideal plastic or Bingham plastic
        - → Dilatant



Ex

τ	0	1.2	2.3	4
du/dy	0	2	3	4



Examples of Non Newtonian

1) Pseudoplastic : (B=0, n<1)

Milk, Blood, pulp paper sol^n, Liq^n cement

2) Ideal (or) Bingum plastic

Drilling mud, sewage sludge, Hyach, toothpaste
(RRB-

NOTE: For shear thickening fluid, viscosity increases with increasing in time. [with elapse of time]

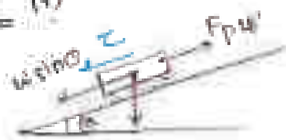
Ex. Find μ, F_{pull} (?)

$\rightarrow \theta = 22.61$
$\phi = 67.38$

$F = A\tau = \mu\left(\frac{du}{dy}\right)A$

$130 \sin 22.61 = \mu\left(\frac{5}{0.0005}\right)\times 1 \Rightarrow$ $\boxed{\mu = 5\times10^{-3}\ \frac{Ns}{m^2}} = 5\ cP$

F_{pull} (?)



$F_{pull} = W\sin\theta + \frac{\mu A V}{y}$

$= 130 \sin 22.61 + \frac{5\times10^{-3}\times1\times5}{5\times10^{-4}}$

$\boxed{F_{pull} = 100\ N}$

→ **Methods to find Viscosity:**

① **Newtonian Equation**

$$\boxed{\tau = \mu\left(\frac{du}{dy}\right)}$$

IMP ② **Hazen Poiseuille's Equation**

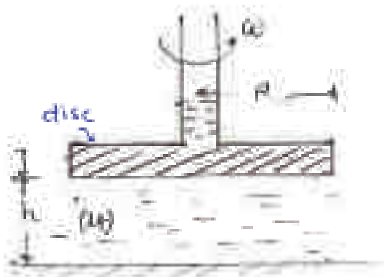
$$\boxed{Q = \left(-\frac{\partial P}{\partial x}\right)\frac{\pi R^4}{8\mu} = \frac{(P_1-P_2)\pi D^4}{128\mu L}}$$

(3) Stoke's Equation :

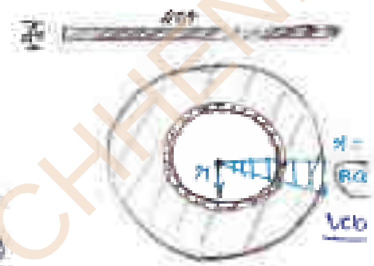
(In stokes eq^n we consider only F_G, F_P, F_V are considered)

$$\text{Velocity} = \frac{gd^2}{18\mu}\left[\rho_s - \rho_L\right]$$

GATE CIVIL



Q^n A circular disc of diameter d was rotating above the fixed surface as shown find, uniform torque required to maintain angular velocity ω (S)



→ $T = \text{Force} \times \text{dist}$
$= F \times R$
$= \frac{\mu A V}{y} \times R$

Here ; $\mu = \mu$
$A = \pi R^2$
$V = R \cdot \omega$
$y = h$

$= \frac{\mu \pi R^2 \cdot (R\omega)}{h} \times R$

$$T = \frac{\pi \mu \omega R^4}{h}$$

(Here we can't dive do that type here. entirely velocity at different points is varies on disc ∴ to take integration.)

$$T = \int dT = \int dF \times r$$
$$= \int \left(\frac{\mu\, dA\, V}{y}\right) r = \int_{r=0}^{R} \frac{\mu (2\pi r\, dr)(r\omega)\, r}{h}$$
$$= \frac{2\pi\mu\omega}{h}\int_0^R r^3\, dr = \frac{2\pi\mu\omega}{h}\left[\frac{r^4}{4}\right]_0^R$$

$$\boxed{T = \frac{\pi\mu\omega R^4}{2h}}$$ Ans - (a) $T = \frac{\pi\mu\omega d^4}{32h}$

→ In previous qⁿ w/o integrating the answer is half of the answer from w/o integration be an

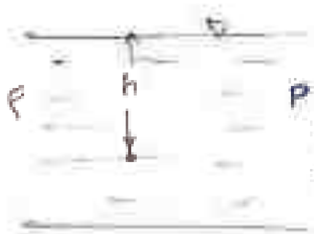
$\boxed{\frac{1}{2}}$

Direct sol^{n} - c
Integration

$\boxed{\frac{2}{3}}$

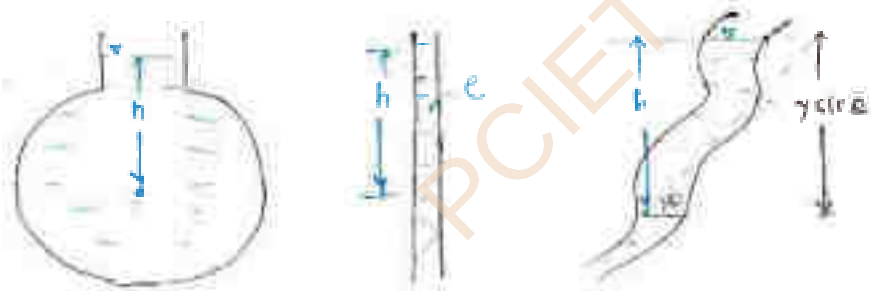
Hydrostatics

➡ Hydrostatic Law :

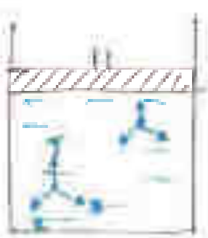


$P = \rho g h = \gamma h$
$= S\rho_w g h$
$= S\gamma_w h$

shapes doesn't matter Height req^{n} (vertical) (measured from vertical)

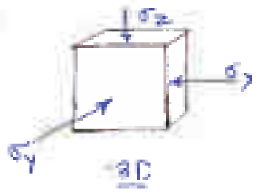


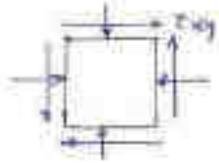
Hydrostatic condition (pascal's law;



Sol :- pressure gives equally propagates in all three direction. (xyz) only in static condition

Hydrostatic





$\sigma_x = \sigma_y = \sigma_z = -\sigma$

$\tau_{xy} = 0$

for 2-D $\Rightarrow$ $\sigma_x = \sigma_y = \sigma$
$\tau_{xy} = 0$

GATE 05 For static condition $\rightarrow$ shear stress $= 0$
Normal stress $= \sigma$

Mohr's Circle :

$$\text{Radius} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \Rightarrow R = \sqrt{\left(\frac{-\sigma-(-\sigma)}{2}\right)^2 + 0} = 0$$

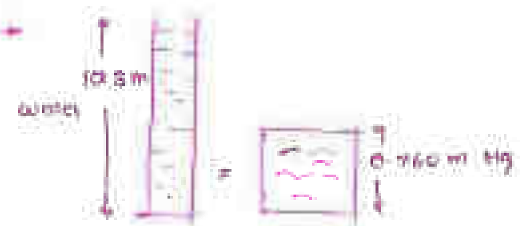
$$\text{Center} = \left[\frac{\sigma_x + \sigma_y}{2}, 0\right] \Rightarrow C = \left[\frac{-\sigma+(-\sigma)}{2}, 0\right] = [-\sigma, 0]$$

Atmospheric Condition :

$P_{atm} = 1$ atm
$= 1.01325$ bar
$= 1.01325 \times 10^5$ N/m²
$= 760$ mm of Hg
$= 10.3$ m of water
$= 760$ torr

M.S.L
[Mean sea level]

$\boxed{1 \text{ torr} = 1 \text{ mm of Hg}}$



Pressure created by 10.3 m water is same as p exerted by 0.760 m Hg is 1.01325 bar

Explanation : $P = \rho g h = 1.013 \times 10^5 \ N/m^2$

$\rightarrow h_{m \text{ of water}} = \dfrac{1.013 \times 10^5}{\rho_w \times g} = \dfrac{1.013 \times 10^5}{1000 \times 9.81} \approx 10.3$ m of water

$\rightarrow h_{m \text{ of Hg}} = \dfrac{1.013 \times 10^5}{\rho_{Hg} \times g} = \dfrac{1.013 \times 10^5}{13600 \times 9.81} \approx 760$ mm of Hg

$\rightarrow h_{m \text{ liq}, s=0.8} = \dfrac{1.013 \times 10^5}{0.8 \times 1000 \times 9.81} \approx 12.8$ m of liq $s = 0.8$

ES q^n 500 mm of Hg ______ m of water

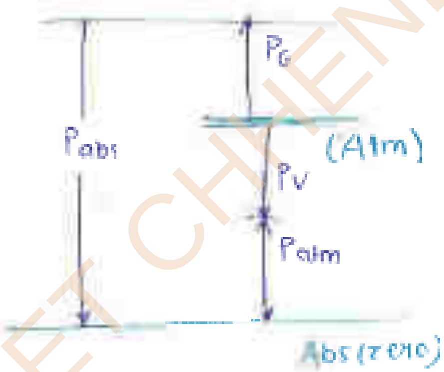
$\Rightarrow \left[\dfrac{1.013 \times 10^5}{760}\right] \times 500 \ N/m^2$

$\Rightarrow \left[\dfrac{10.3}{760}\right] \times 500$ m of water

⇒ Gauge pressure (P_G) :-

— pressure measured above atmosphere

+ve P_G



⇒ Vaccum Pressure (P_V) :

— below atmosphere

−ve P_G

— suction pressure

⇒ Absolute pressure (P_{abs})

$P_{abs} = P_{atm} + P_G - P_V$

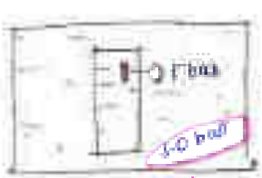
NOTE → The pressure of atmosphere in which the gauge is located

Q

$P_b = P_{atm} + P_G$

$= 1.01 + 5$

$P_b = 6.01$ bar (abs)

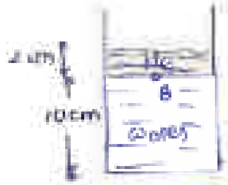


$P_{abs} = P_{atm} + P_G$

$P_{atm} = 6.0$ bar

this is atm pressure for cyl.

ES A cylindrical container contains water to a height of 10 cm above that mercury is adding to ht of 2 c[m] then the pressure at the interference at equilibrium



$\cancel{P_h = \rho_{Hg}\, g\, h = 13600 \times 9.81 \times 0.02}$

Here Hg have higher density so,

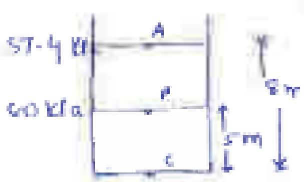
$P_h = \rho_w g h$
$= 1000 \times 9.81 \times 0.1$
$= 981\ N/m^2$

NOTE

$\rho_1 > \rho_2$

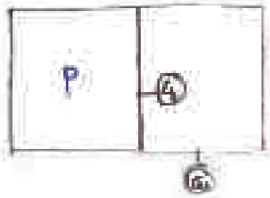
$P_A = P_{atm}$ $\boxed{\rho_1 > \rho_2}$
$P_B = P_A + \rho_2 g h_2$
$P_C = P_B + \rho_1 g h_1$
$= P_A + \rho_2 g h_2 + \rho_1 g h_1$

GATE CIVIL A pressure gauge reads 57.4 kPa & 80 kPa respectively and heights of 5 m & 8 m then fixed on side of tank filled with liquid then approximate density of liquid will be ?



$P_1 = 57.4 \times 10^3\ N/m^2$
$P_1 = \rho g h_1$; $P_2 = \rho g h_2$
$P_2 - P_1 = \rho g h_2 - \rho g h_1$
$80 - 57.4 = \rho g (h_2 - h_1) = \rho g (8 - 5)$
$\frac{22600}{9.81 \times 3} = \rho$
$\boxed{\rho = 767.9\ kg/m^3}$

GATE



$P_{G_1} = 5.0$ bar

$P_{G_2} = 2.0$ bar

$P_{atm\ std} = 1.01$ bar

$P_{abs} = ?$

$\rightarrow$ $P_{abs} = \underline{P_{atm(G_1)}} + P_{G_1}$

$[P_{atm(G_1)}]_{abs} = P_{atm(G_2)} + P_{G_2}$

$= 1.01 + 2.0$

$= 3.01$

$\rightarrow$ $P_{abs} = 3.01 + 5.0$

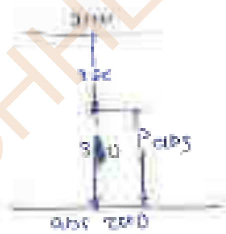
$= 8.01$ bar

ES Std. atm. pressure 760 mm of mercury at a specific location barometer reads 700 mm of Hg, what does an absolute pressure of 380 mm of Hg at this location refer to ?

$\rightarrow$ $P_{abs} = P_{atm} \pm P_G/P_V$ (local atm pressure)

$380 = 700 + P_G/P_V$

$\boxed{P_V = -320 \text{ mm of vacuum}}$



if P_V is negative then vaccum
P_G is positive then gauge

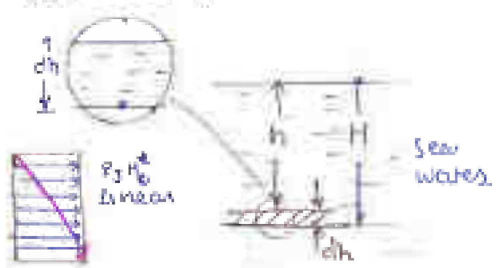
ES Density of sea water varies with depth as $\rho = \rho_0[1+h]$, then the pressure at depth of H from the free surface will be ?

$\rightarrow$ $\int dp = \int \rho g\, dh$ $\quad \int dp = \rho g\, dh$

$\rho = \rho_0[1+h]$ given

$\rho = \rho_0[1+H] \times g \times H$

$= \rho_0 g H + \rho_0 g H^2$



$P = \int dP$, $dP = \rho_0 (1+h) g \times dh$

$$P = \int_{h=0}^{H} \rho_0 (1+h) g \, dh = \rho_0 g \left[h + \frac{h^2}{2}\right]_0^H$$

$$\boxed{P = \rho_0 g H + \frac{\rho_0 g H^2}{2}}$$ Ans

Note → $\gamma = \gamma_0 + C\sqrt{h}$.

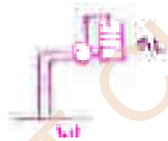
$P @ H \Rightarrow P = \gamma_0 H + \frac{2}{3} C H^{3/2}$

⇒ Vapour pressure

Cavitation (to avoid) cavitation P must be more than the vapour pressure $[P > P_v]$

$$\boxed{P_{min} > P_{vapour}}$$

	Temp	pressure
↑	100 °C	10.3 m of water ↑
↓	30 °C	2.36 m of water ↓



$Q = AV$ (const, const)

$z + \frac{P}{\rho g} + \frac{V^2}{2g} = C$ (const)

$Q = (A\ V\uparrow)$

$z + \downarrow\frac{P}{\rho g} + \frac{V^2}{2g}\uparrow = C$

→ $P_{min} > P_{vapour}$

$\sigma > \sigma_c$

$$\frac{NPSH}{H} > \sigma_c$$

σ = thoma no.

NPSH = Net positive suction head

σ_c = critical 'σ'
= cavitation coefficient

GATE

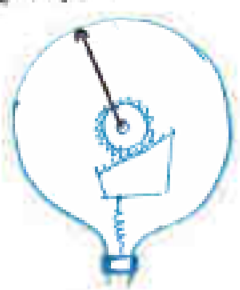
Activity	→	property
→ Lubrication	→	viscosity
→ formation of spherical droplet	→	surface tension
→ Rise of sap in trees	→	capillarity
→ Cavitation	→	vapour pressure
→ Hammering effect	→	valve closure

to avoid ($P_{min} > P_{vap}$)

to avoid (surge tank)

⇒ Pressure Measurement

Bourden pressure gauge



(static) condition

→ $P \times A = K \times \delta$

→ pressure transducer

(It's not static)

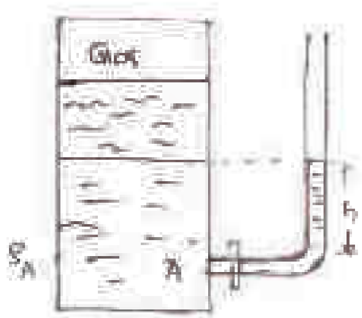
→ It is of peak pressure in milli second only so, then it will decrease

→ It measure in 3-4 ms time

⇒ Piezometer

(measure for pressure)

↳ It measure gauge pressure only



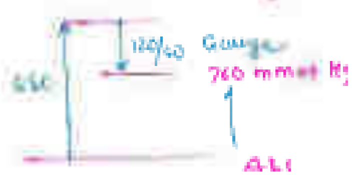
$P_A = \rho_A g h$

→ For measurement of Blood pressure

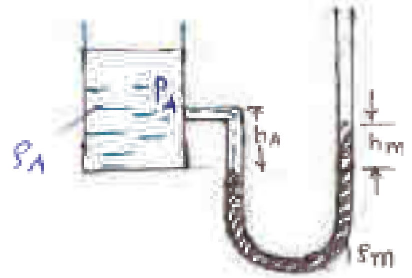
Sigma manometers

120/80) mm of Hg

↳ is gauge pressure



⇒ Simple U-tube Manometer :- 1^{st} method
[lowest level Ref.]



→ $P_A + \rho g h_A = \rho_m g h_m + P_{atm}$

P_A ↓ gauge pressure

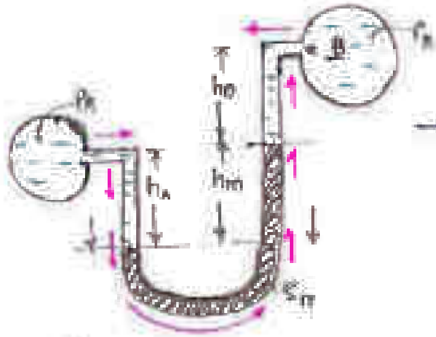
(P_{atm} open to air)

NOTE :- When it is open to the air at that time atmospheric pressure ...

$P_{atm} = 0 \Rightarrow P_A = P_{gauge}$

$P_{atm} \neq 0 \Rightarrow P_A = P_{abs}$

⇒



→ $P_A + \rho_A g h_A = P_B + \rho_B g h_B + \rho_m g h_m$

⇒ 2^{nd} Method

$P_A + \rho_A g h_A - \rho_m g h_m - \rho_B g h_B - P_B = 0$

Property :

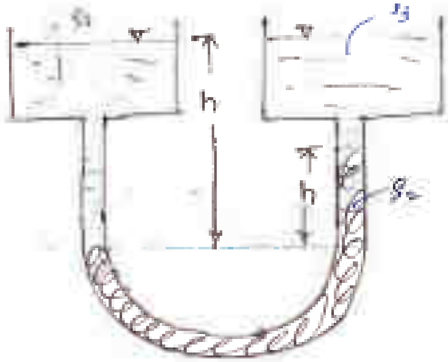
(1) Vapour pressure (low)

(2) ρ, S, γ → high

(cavitation / evaporation — vapour p. as low as possible)

Expansion of tube, where friction & acceleration

Es



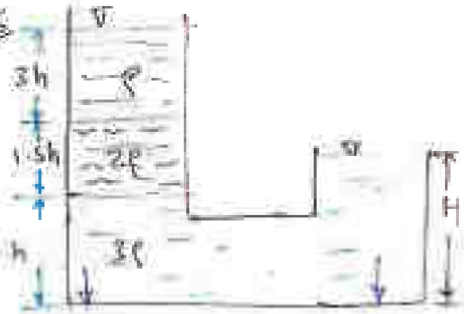
$$P_{atm} + \rho_1 g h_1 = P_{atm} + \rho_3 g [h_1 - h] + \rho_2 g h$$

$$\rho_2 g h - \rho_3 g h = \rho_1 g h_1 - \rho_3 g h_1$$

$$h[\rho_2 - \rho_3] = h_1[\rho_1 - \rho_3]$$

$$\boxed{h = \left[\frac{\rho_1 - \rho_3}{\rho_2 - \rho_3}\right] h_1}$$ ans

Es

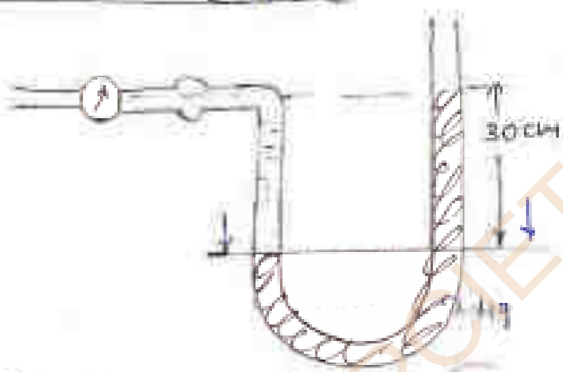


$$\rho(3h) + 2\rho(1.5h) + 3\rho(h) = (3\rho)(H)$$

$$9h = 3H$$

$$\frac{H}{h} = 3$$

ISRO-10 +16



The pressure 'P' in kPa (?)

(a) 25.3
(b) 38.5 ✓
(c) 43 ×
(d) 55 ×

$P_{ab} = P_{atm} + P_g$
$= 10^5$ ...

(is not this given above here)

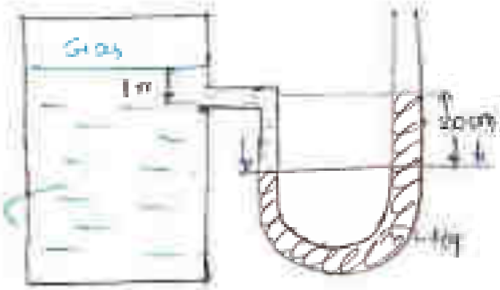
$$P_a + \rho_w g h_w = \rho_{Hg} g h_{Hg} + P_{atm}$$

$$P + 1000 \times 9.81 \times 0.3 = 13600 \times 9.81 \times 0.3$$

$$= 9.81 \times 0.3\,(13600 - 1000)$$

$$\boxed{P = 37.08 \text{ kPa}}$$

IAS-05



Pressure of gas in m of water is

(a) 2.72
(b) 2.52
(c) 1.72
(d) 1.52 ✓

$P + \rho g(1) + \rho g(0.2) = \rho g(0.2)$

$P + 1000 \times 9.81 \times 1.2 = 13600 \times 9.81 \times 0.2$

$= 9.81(2720 - 1200)$

$= 14911.2 \ N/m^2$

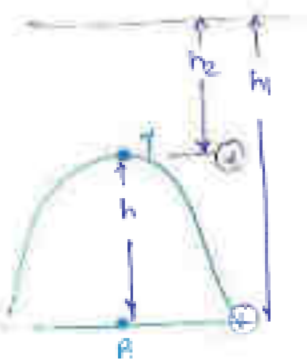
1.0132 bar ⟶ 10.3 m of water

$101325 \ N/m^2$ ⟶ 10.3 m of water

$14911.2 \ N/m^2$ ⟶ (?)

1.51 m of water

ES The pressure at the bottom of the mountain $P_B = 700$ mm of Hg & pressure at top of mountain $P_A = 600$ mm of Hg. Considering the constant density of air 1.2 kg/m³ the approximate height of mountain (?)



$P_B = 700$ mm of Hg $= 0.9210$ bar

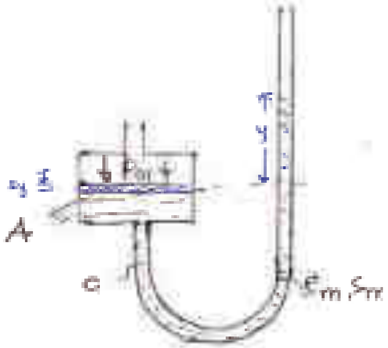
$P_A = 600$ mm of Hg $= 0.7894$

$P_B - P_A = \rho g(h_2 - h_1)$

$(1.01 \times 10^5)(0.1316) = (1.2)(9.8)(h_2 - h_1)$

$h_1 - h_2 = 1.119$ km

⇒ Single column U-tube Manometer [Micromanometer]



$$\boxed{\frac{a}{A} = \frac{1}{100} \text{ to } \frac{1}{500}}$$

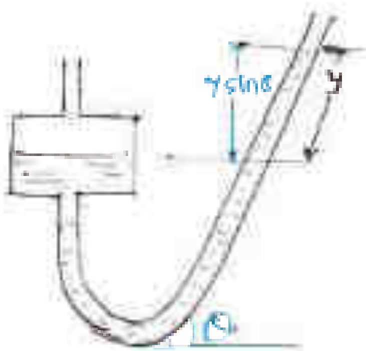
$$A\,\Delta y = a\,y$$

$$\frac{a}{A} = \frac{\Delta y}{y}$$

$$\boxed{P_G = \rho_m g y}$$

$$= S_m \rho_\omega g y$$

$$\boxed{P_G = \gamma_\omega S_m y}$$



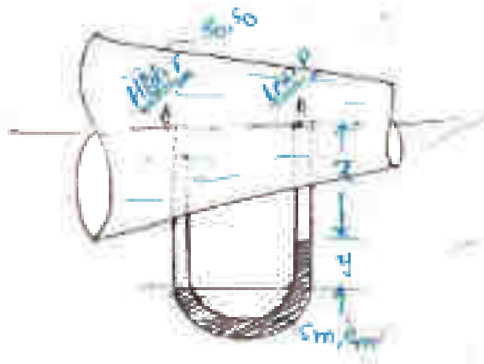
$$\boxed{P_G = \gamma_\omega S_m y \sin\theta}$$

⇒ sensitivity (↓) → θ (↑)

$$\boxed{\underset{(S)}{\text{sensitivity}} = \frac{1}{\sin\theta}}$$

where θ = 0° to 90°
sinθ = 0 to 1

Differential U-tube Manometer :-



$P_A + \rho_o g(x+y)$
$= P_B + \rho_o g x + \rho_m g y$

$P_A - P_B = \rho_m g y - \rho_o g y$

$$\boxed{P_A - P_B = (\rho_m - \rho_o) g y}$$

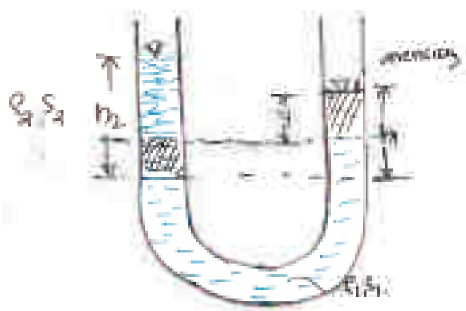
$= [S_m \rho_w - S_o \rho_w] g y$

$P_A - P_B = \rho_w g y [S_m - S_o]$

$P_A - P_B = \gamma_w y [S_m - S_o]$ N/m^2

$\frac{P_A - P_B}{\rho g} = \frac{\gamma_w y [S_m - S_o]}{\rho_w g} = y[S_m - S_o]$; m of water

$\frac{P_A - P_B}{\rho_o g} = \frac{\gamma_w y [S_m - S_o]}{S_o \rho_w g} = y\left[\frac{S_m}{S_o} - 1\right]$; m of liqⁿ of (S_o) flowing fluid



① What is the deflection above initial level → h_1

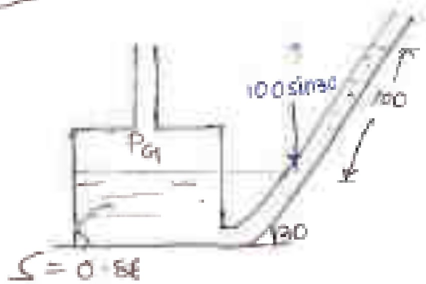
② Diffⁿ of levels in both the liquid (?) → h_2 h

③ Rise of meniscus → z
$h_1 = y_1 + y_1 \Rightarrow \boxed{y = h_1/2}$

$P_{atm} + \rho_2 g h_2 = P_{atm} + \rho_1 g h_1$

$\Rightarrow \boxed{\therefore \rho_1 h_1 = \rho_2 h_2}$

GATE-07
CIVIL



$y = 100 \sin 30$
$= 50$

$P_G = \rho_0 g y$
$= 0.86 \times 10^3 \times 9.81 \times 50$
$= \frac{421.83}{9.81}$
$= 43$ mm of water (Gauge)

(If towards atmosphere so gauge, if against atm then vacuum

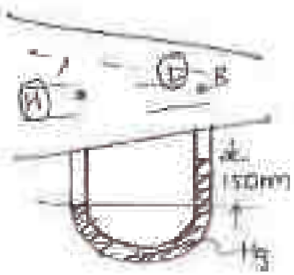
→ $P_G = S_m \gamma_w y \sin\theta$ (N/m^2)

$\frac{P_G}{\gamma_w} = S_m y \sin\theta$

$= 0.86 \times 100 \times \sin 30$

$= 43$

GATE-05



a) flow dir^n A → B, $P_A - P_B = 20$ kPa
b) " " " $= 1.4$ kPa
c) flow dir^n B → A, $P_B - P_A = 20$ kPa
d) " " " $= 1.4$ kPa

(It should be A → B or B → A but if dia is const then only A → B

$P_A - P_B = (\rho_m - \rho_0) g \, y$

$= (13600 - 1000) \times 9.81 \times 0.15$

$= \underline{20 \text{ kPa}}$

A Differential mercury manometer used to measure water flow reads 20 cm the corresponding pressure head difference in ___ m of water.
(a) 2.72 (b) 2.52 (c) 1.52 (d) 1.72

$\frac{(P_A - P_B)}{\rho_w g} = y(S_m - S_0)$

$= 0.2(13.6 - 1)$

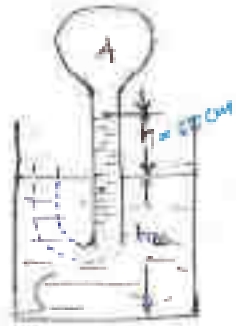
$= \underline{2.52}$ m of water

Ex-14 A 6 cm column of liquid 'A' was balancing 3 cm column of liquid 'B'. $\frac{\rho_A}{\rho_B} = (?)$

$\rightarrow S_A h_A = S_B h_B$

$$\frac{S_A}{S_B} = \frac{\rho_A}{\rho_B} = \frac{h_B}{h_A} = \frac{3}{6} = \boxed{1/2}$$

GATE 2008



pressure of Gas in bulb A:
$P_A = 50$ cm of Hg vacuum
$P_{atm} = 76$ cm of Hg
then $h = (?)$

(a) 26 cm (b) 50 cm
(c) 76 (d) 126 cm

$\rightarrow P_{atm} = P_A + \rho_{Hg} g h$ N/m^2

$$\frac{P_{atm}}{\rho_{Hg} g} = \frac{P_A}{\rho_{Hg} g} + h$$

$P_{atm} = 0 \rightarrow P_A$ (gauge pressure)
$\neq 0 \rightarrow P_A$ (absolute P) is given

$$(P_A)_{abs} = P_{atm} - P_V$$
$$= 76 \text{ cm} - 50 \text{ cm}$$
$$= 26 \text{ cm}$$

$h_w = S_{liq} h_{liq}$
$= 13.6 \times 0.5$
$= 6.8$ m water

$\rightarrow 76 = 26 + h \Rightarrow \boxed{h = 50 \text{ cm}}$

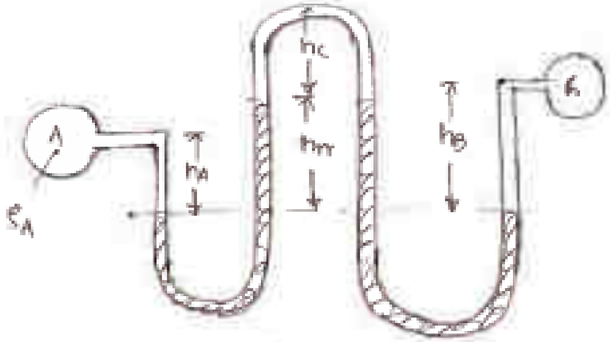
NOTE In problem $P_c = 5$ m water of vacuum is given then 5 m is height of its suction

$P_c = 5$ m of water vacuum

$P_c + \rho_w g 5 = P_{atm}$
$P_c = P_{atm} - \rho_w g 5$

$\rightarrow P_B = 5$ m of water [Gauge]

GATE-16
IAS 08



$P_A - P_B = ?$ m of water

$$P_A + \rho_A g h_A + \rho_m g h_m = \rho_m g h_m + \rho_B g h_B + P_B$$

$$\frac{P_A - P_B}{g} = (\rho_B h_B - \rho_A h_A)$$

for m of water unit P is divide with $\rho_w g$

$$\frac{P_A - P_B}{\rho_w g} = S_B h_B - S_A h_A \text{ m of water}$$

$$= \left[\frac{S_B h_B - S_A h_A}{S_A}\right] \text{ m of liq}^n \text{ A}$$

$$= \left[\frac{S_B h_B - S_A h_A}{S_B}\right] \text{ m of liq}^n \text{ B}$$

ES OMC An open, U-tube with uniform bore of 0.5 cm^2 with one of the limb vertical & other inclined at 30° with horizontal with initially filled with liquid of S.G. 1.25 & and additional 7.5 cc of water was added in inclined limb the rise of mercury in vertical limb will be (?)

a) 8 cm
b) 6 cm
c) 4 cm
d) 3 cm

$V = 7.5\ cm^3$, $A = 0.5$

$h_w = 15$ cm

$$P_{atm} + \rho_2 g h_2 = P_{atm} + \rho_1 g h_1$$

$$S_2 h_2 = S_1 h_1$$

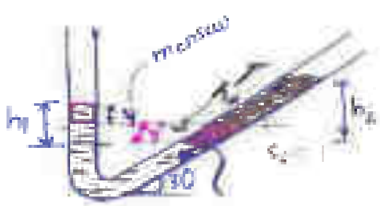
$$(1.25)(15 \sin 30) = (1)(h_1)$$

$$S_1 h = S_2 h_2$$

$$1.25 \times h_1 = 1 \times 7.5$$

$$\boxed{h_1 = 6\ cm}$$

rise mercury $y = \frac{h_1}{2} = \frac{6}{2} = \boxed{3\ cm}$ X



Total movement $h_1 = y + y$
In inclined ⇒ $h_1 = y + y'$ ⇒ $6 = y + y \sin 30$ ⇒ $\boxed{y = 4}$

Hydrostatic Pressure

① Horizontal plate / plane / lamina

Hydro. st. pr. force $\boxed{\bar{h} = \bar{x}}$

$F = P * A$

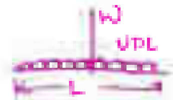
$= \gamma h * A$

$= \rho g h A$

$\boxed{F = \gamma A \bar{x}}$

$\left[\because \gamma = \rho g,\ h = \bar{x}\right]$

center of pressure



Location	F	$\bar{h}$
	$\gamma A \bar{x}$	$\bar{x}$
 C.G. / C.P.	$\gamma A \bar{x}$	$\bar{x} + \frac{I_G}{A\bar{x}}$
	$\gamma A \bar{x}$	$\bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$

② Vertical plate

$F = P \times A$

$\int dF = \int P\,dA = \int \gamma h\,dA$

$= \gamma \int h\,dA$

$= \gamma \int h\,dA$

$\boxed{F = \gamma A \bar{x}}$

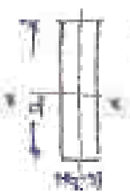
$\int y\,dA = A\bar{x}$

$\int y^2\,dA = I_G + A(\bar{x})^2$

$\boxed{\bar{h} = \bar{x} + \left(\frac{I_G}{A\bar{x}}\right)}$

where: I_G = moment of inertia about centroid

Moment of Inertia : (I_G)



$I_{xx} = \frac{bd^3}{12}$

$I_{yy} = \frac{db^3}{12}$

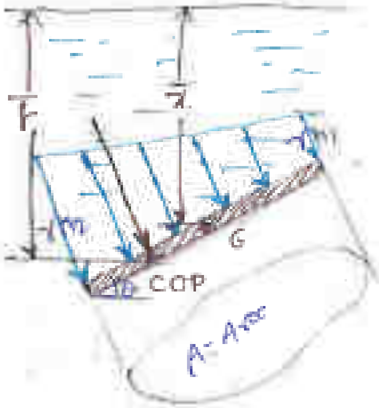
$\frac{\pi(d_o^4 - d_i^4)}{64}$

$I_{xx} = 0.11 R^4$

$I_G = 0.035\pi$

$h = \frac{4R}{3\pi}$

③ Inclined plane

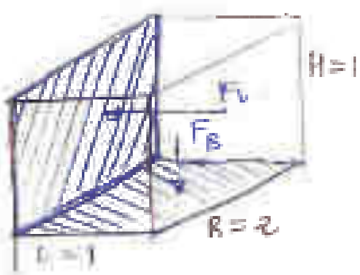


$F = \gamma A \bar{x}$

$$\boxed{\bar{h} = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}}$$

ES-15) An open rectangular container with L : B : H = 1 : 2 : 1 was completed filled with water, the ratio of hydrostatic force acting at bottom face to force acting on any one of its larger vertical face is

$\frac{F_B}{F_V} = ?$



$F_B =$
$= \gamma A \bar{x}$

$A_B = (2 \times 1) = 2$
$\bar{x}$ = Dist^n from free surface = 1

$F_B = \gamma A \bar{x}$
$\boxed{F_B = 2\gamma_w l}$

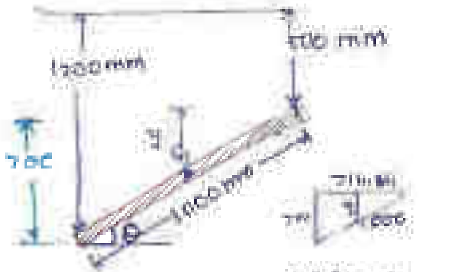
F_V
$= \gamma A \bar{x}$

$F_V = \gamma A \bar{x}$
$= \gamma_\omega \times 1 \times 2 \times \frac{1}{2}$

$\boxed{F_V = \gamma_\omega}$

$A_V = 1 \times 2$
$\bar{x} = \frac{1}{2}$

GATE 01 → A circular lamina of 1000 mm in diameter, was lying in water such that the distance of its perimeter measured vertically below the free surface is varying from 500 mm to 1200 mm. Find $F, \bar{h} = ?$



$$\frac{700}{714.4} = \frac{y}{357.01}$$

$\rightarrow F = \gamma A \bar{x}$

$\gamma = \rho g = 9810 \ N/m^2$

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} \ m^2$

$y = 350 + 500 = 850$

$\bar{x} = 0.85$

$F = 9810 \times \frac{\pi}{4} \times 0.85$

$\boxed{F = 6.54 \ kN}$

$\sin\theta = 7/10$
$= 0.7$

$$h = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$$

$$= 0.85 + \frac{\left(\frac{\pi}{64} \times 1^4\right) \sin^2\theta}{\frac{\pi \times 1^2}{4} \times 0.85} \quad (0.7)^2$$

$\boxed{h = 0.886 \ m}$

03. A vertical rectangular dock gate of 3 m wide & 5 m height was closing a tunnel running full with water the pressure at bottom of the gate of 195 kN/m². The hydrostatic pr. force acting = ?

→ $F = \gamma A \bar{x}$, $A = 3 \times 5 = 15 \ m^2$

$\bar{x} = 5/2$

$\gamma = \gamma_{w}$

$F = \rho_w g A \bar{x}$

$= 1000 \times 9.81 \times 15 \times 2.5$

$= 367.87$ Here 195 kN/m² at bottom given.

$\bar{x} = 19.87 - 2.5 = 17.37$

$\bar{x} = h_B - 2.5$

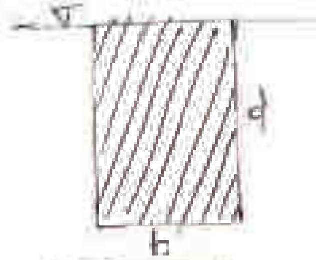
$P_B = \rho_w g h_B \Rightarrow h_B = \frac{195 \times 10^3}{9.81 \times 10^3}$

$\boxed{h_B = 19.87 \ m}$

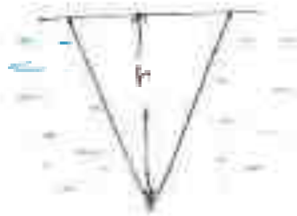
$F = 9810 \times 3 \times 5 \times 17.37$

$\boxed{F = 2.55 \ MN}$

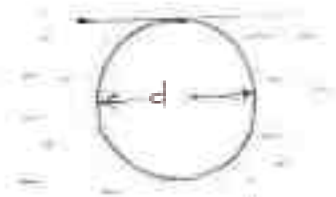
Standard cases of center of pressure

① 

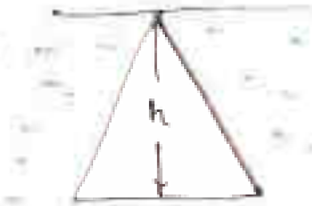
$$\boxed{\bar{h} = \frac{2d}{3}}$$

② 

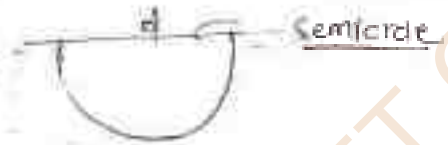
$$\boxed{\bar{h} = \frac{h}{2}}$$

③ 

$$\boxed{\bar{h} = \frac{5}{8}d}$$

④ 

$$\boxed{\bar{h} = \frac{3}{4}h}$$

⑤ 

$$\boxed{\bar{h} = \frac{3\pi d}{32}}$$

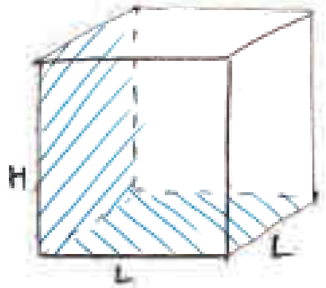
Note → 

$$\bar{h} = \cancel{c + \frac{2d}{3}}$$

$$\bar{x} = c + \frac{d}{2}$$

$$\bar{h} = \left(c + \frac{d}{2}\right) + \frac{\frac{bd^3}{12}}{(bd)\left[c + \frac{d}{2}\right]}$$

Ex. Rectangular container with equal vertical sides of $L \times H$, completely filled with water; if $F_{Bottom} = F_{vertical}$, $L/H = ?$



$F_{Bottom} = F_B$

$F_B = \gamma A \bar{x}$
$= \gamma_w (L \times L)(H)$

$F_{vertical} = F_v$
$F_v = \gamma A \bar{x}$
$= \gamma_w (L \times H)(H/2)$

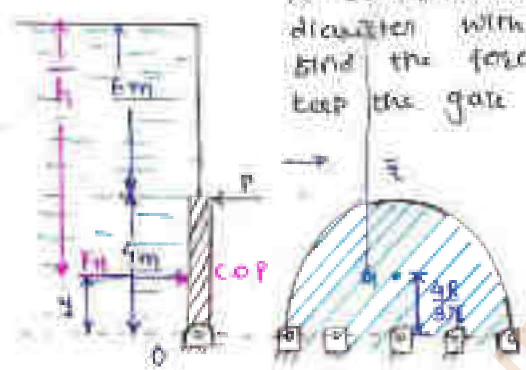
$F_B = F_v$

$\gamma_w (L \times L) H = \gamma_w (L \times H)(H/2)$

$\boxed{L/H = 1/2}$

Ex

A semicircular gate hinged along its diameter with water on its side as shown. Find the force required at point P to keep the gate vertically.



$\Sigma M_O = 0$

$F_P \times 4 = F_H \times y$

$F_P \times 4 = (\gamma A \bar{x}) \times y$

$F_P = \dfrac{\gamma A \bar{x} y}{4}$

Now, $\gamma = 9810 \ N/m^3 = \rho g$

$A = \dfrac{\pi R^2}{2} = \dfrac{\pi 4^2}{2} \ m^2$

$\bar{x} =$ C.G. from free surface

$= 10 - \dfrac{4R}{3\pi} = 10 - \dfrac{4 \times 4}{3\pi} = 8$

but; $\bar{h} = \bar{x} + \left(\dfrac{I_G}{A\bar{x}}\right)$

$= 8.3 + \dfrac{0.055 \pi R^4}{\frac{\pi R^2}{2} \times 8.3}$

$\bar{h} = 8.43$

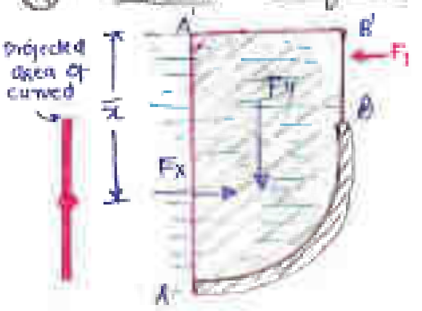
$y = 10 - \bar{h}$
$= 10 - 8.43 = 1.57$

$I_G = 0.11 R^4$

so; $F_P = \dfrac{9810 \times \frac{\pi}{2} 4^2 \times 8.3 \times 1.57}{4}$

$\boxed{F_P = 1160 \ N}$

④ Curved Surface :



$$R = \sqrt{F_x^2 + F_y^2}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right)$$

→ F_y = Vertical Hydrostatic force.

F_y = The weight of liquid in the imaginary volume formed by extending from curved surface till free surface.
= wt. of liq. in (ABB'A'A)

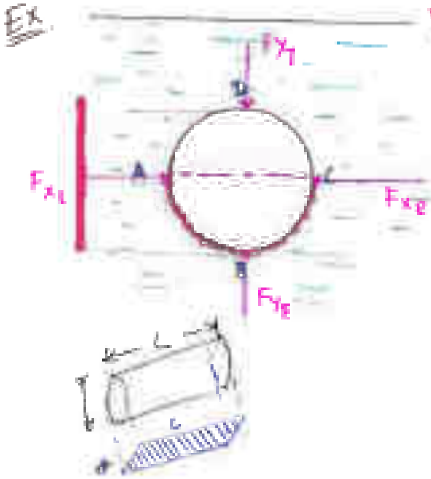
Will be acting through centre of gravity of imaginary vol^n

→ F_x = Horizontal Hydrostatic pressure force.

F_x = Net hydrostatic pressure force on vertically projected area of the curved surface
$= \gamma A\bar{x}$ (∵ A = projected area)

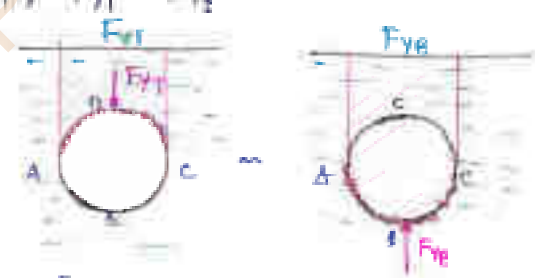
where $\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}}$

Ex



Vertical

$F_y = F_{yT} \sim F_{yB}$



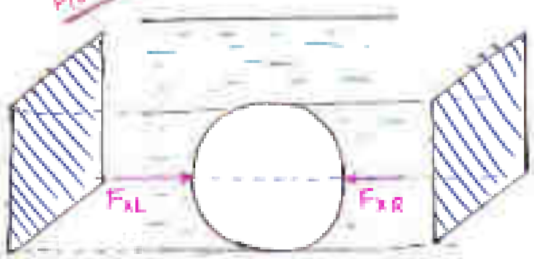
$F_y = F_{yB} - F_{yT}$

F_y = wt. of liq in cylindrical box (...)

$= \gamma \times Vol^m_{cyl}$

$F_y = \rho g \times \frac{\pi}{4} d^2 \times L$

Horizontal



$F_x = F_{xL} - F_{xR} = 0$

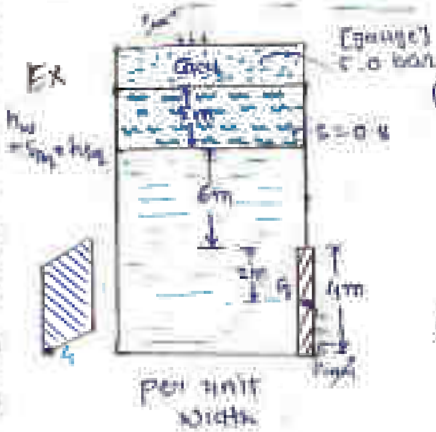
$F_x = 0$

$R = \sqrt{F_x^2 + F_y^2}$

so, F_y = wt. of liq^n in cylinder box

$$\boxed{R = \rho g \frac{\pi}{4} d^2 L = F_y}$$

Ex



① $F = \gamma A \bar{x}$ (open to air)

wh- $\gamma = \gamma_w = 9810\ N/m^3$

$A = 4 \times 1 = 4\ m^2$

$\bar{x} = 6 + 2 = 8\ m$

$$\boxed{F = 313.9\ N}$$

② Added s = 0.8 above free surface (open to air)

$F = \gamma A \bar{x}$

wh- $A = 4 \times 1 = 4\ m^2$

$\gamma = \gamma_w = 9810\ N/m^3$

$\bar{x} = \underbrace{(2+6)}_{water} + 5 = 13$ ×

Eq → $\bar{x} = \underbrace{8}_{water} + (0.8 \times 5) = 12$

we convert other liq^n in water: $S_1 h_1 = S_2 h_2$, $h_w = S_{liq} \cdot h_{liq}$

③ Gas of P = 5.0 bar [gauge]

$F = \gamma A \bar{x}$

$\gamma = \gamma_w = 9810\ N/m^3$

$A = 4 \times 1$

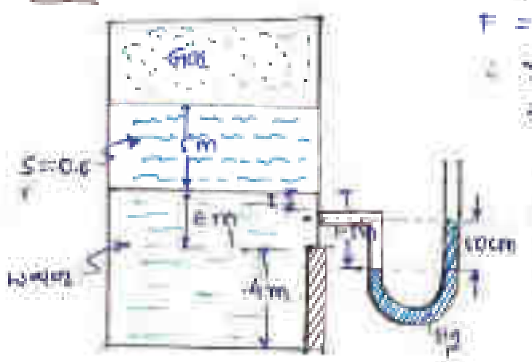
$\bar{x} = \underbrace{(2+6)}_{w} + \underbrace{5}_{(S=0.8)} + \underbrace{h_G}_{(gas)}$

$= \underset{w}{8} + \underset{w}{(0.8 \times 5)} + \underset{w}{\left(\frac{10.3}{1.013}\right) \times 5.0}$

1.013 bar → 10.3 m of w

5.0 bar → ?

GATE



For find ...

$F = \gamma A \bar{x}$

$\because \gamma = \gamma_w = 9810 \ N/m^3$

$\bar{x} = (6+2)_w + (5)_{s=0.6} + P_{gau}$

For find pressure

$p = \rho g h$

$= 13.6 \times 9.81 \times 0.5$

$P_{atm} + P_G = P_{atm}$

$\rightarrow P_G + (5)_{s=0.6} + (1.5)_w = (0.5)_{Hg} \rightarrow$

so $\bar{x} = (6)_w + (5)_{s=0.6} + P_{gauge}$

$= (6)_w + \underbrace{\{(1.5)_w + (5)_{s=0.6} + P_G\}}_{(0.5)_{Hg}}$

$= (6)_w + (0.5)_{Hg}$

$= (6)_w + (13.6 \times 0.5)_w$

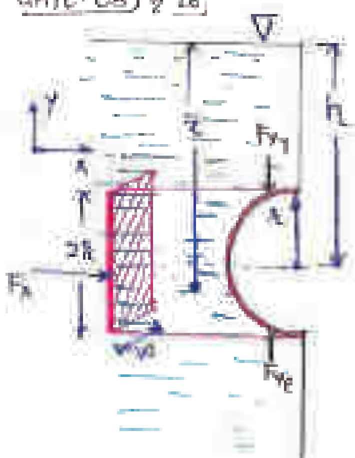
$\boxed{\bar{x} = 13.3 \ m \text{ of water}}$

from $s_1 h_1 = s_2 h_2$

$h_w = S_{liq} \times h_{liq}$

$= 13.6 \times 0.5$

GATE-08) Q 16

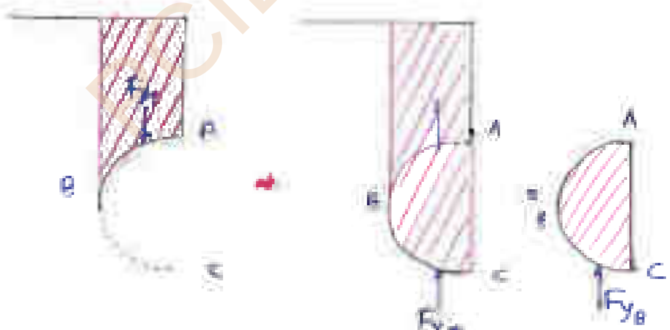


$F_x = \gamma A \bar{x}$

$= (\rho g)[2R \cdot w][h]$

$\boxed{F_x = 2\rho g h R w}$

$F_y = F_{yT} - F_{yB}$



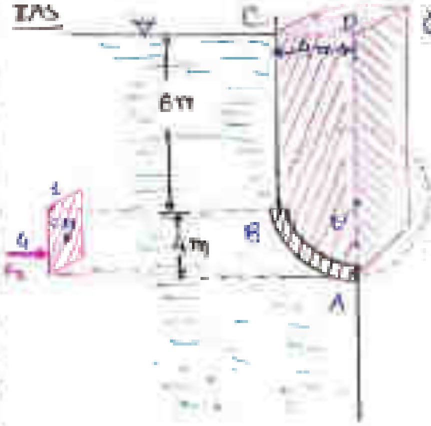
$F_y = F_{yB} - F_{yT}$

$=$ wt. of liq^n in ABC semi cylinder vol^m

$= \gamma \times \text{Vol}_{\text{semi cyl}} = \gamma \times \left(\frac{\pi R^2}{2} \times w\right)$

$\boxed{F_y = \frac{\pi R^2}{2} \rho g W}$

IAS



① F_x :

$A = 4 \times 1$

$\bar{x} = 4 + 2 = 6$

$F_x = \gamma A \bar{x}$

$= \gamma_w \times (4 \times 1) \times (6)$

$= 9810 \times 4 \times 6$

$\boxed{F_x = 313.92 \text{ kN}}$

② F_y :

$F_y = (\text{wt of liq}^n \text{ in imaginary vol}^n)_{ABCDE}$

$= \gamma_w \times (\text{vol}^n)_{img}$

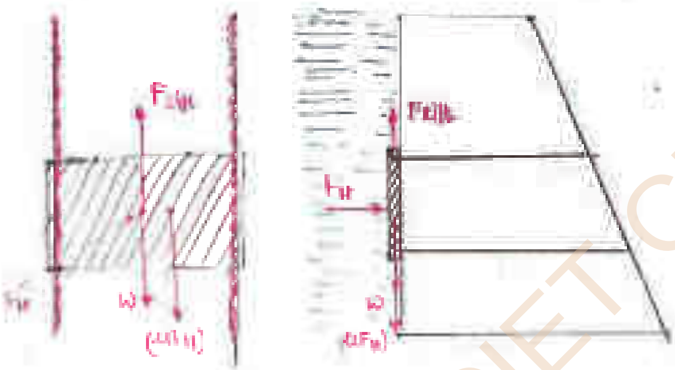
$= \gamma_w [\text{Area} * \text{width}]$

$= \gamma_w \left[\text{(quadrant ABC)} + \text{(rectangle CDE)} \right] * 1$

$= 9810 \left[\frac{\pi \times 4^2}{4} + 4 \times 8 \right] * 1$

$\boxed{F_y = 556.4 \text{ kN}}$

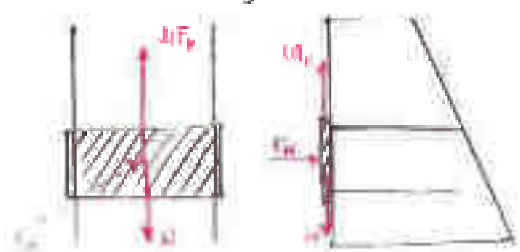
Dockgate Applications :-



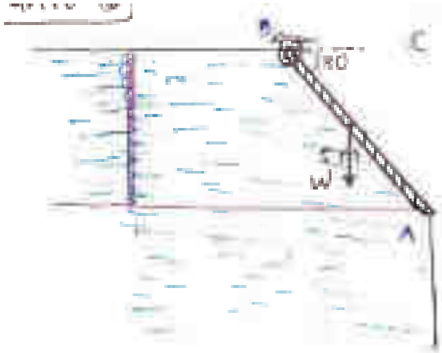
$\boxed{F_{lift} = W + \mu F_H}$

; μ = coefficient of friction

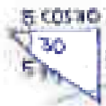
→ Just starting down :



$\boxed{\mu F_H = W}$



A rectangular hinged gate of 5 m length is inclined at 30 with water max on its left as shown. Find the minimum mass of the gate in kg/meter width ⊥ to plane of paper is required to keep it close.



$\frac{1}{2}(2.5 \times 4.33)$

→ $F_x = \gamma A \bar{x}$

$= (\rho g) A \bar{x}$

$= (9.81 \times 1000)(5 \times 1)(2.5)$

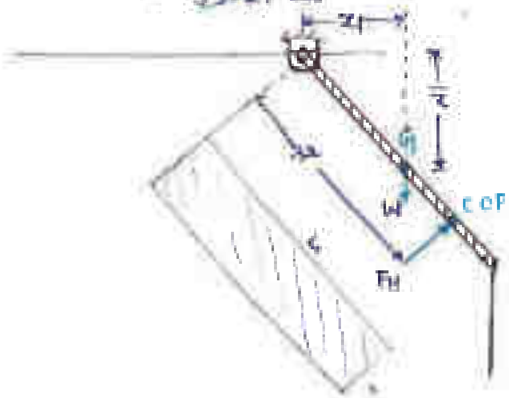
$= 122.625$ kN

→ $F_y = \gamma \times vol^m$

$= \gamma_w \times (vol^m)_{img}$

$= 9810 \times [\ \frac{1}{2} \times 5\cos 30 \times 5 \sin 30\ ]$

$= 53.09$ kN



$\Sigma M_o = 0$

$W x_1 = F_H \times x_2$

$mg x_1 = \gamma A \bar{x} \times x_2$

$\text{mass}(m) = \frac{\gamma A \bar{x} \times x_2}{g x_1}$

Where,

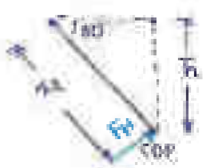
$\gamma = \gamma_w = 9810$ N/m³

$A = 5 \times 1$

$\bar{x} = 1.25$

$x_1 = 2.16$

$x_2 = \frac{1.67}{\sin 30} = 3.34$



$x_2 = \frac{h}{\sin 30}$

by $h = \bar{x} + \frac{I_G \sin^2\theta}{A\bar{x}}$

$= 1.25 + \frac{\frac{1 \times 5^3}{12} \sin^2 30}{5 \times 1 \times 1.25}$

$= 1.667$

$m = \frac{9810 \times 5 \times 1 \times 1.25 \times 3.34}{9.81 \times 2.16}$

$\boxed{m = 9623.1 \text{ kg}}$

GATE - 11 CIVIL

$F = \gamma A \bar{x}$

$F_1 = 11$ kN

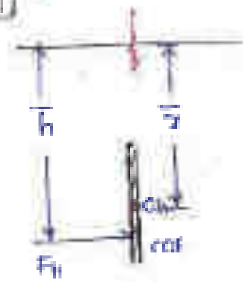
$F = \gamma A \bar{x}$

c c c

$\boxed{F_2 = 11 = F_1}$

($\bar{x}$ from centre mean)

ES - 11



When plate moves down, Difference bet^n C.G & C.O.P will be

(1) ↑ (2) ↓ (3) const (4) x

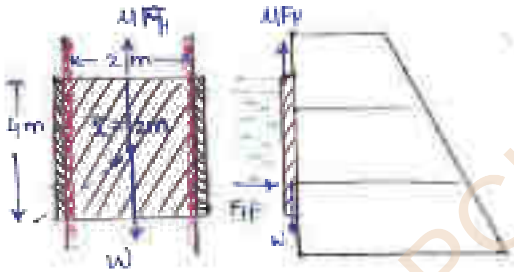
$\bar{x}$ = C.G

$\bar{h}$ = C.O.P

$\bar{h} = \bar{x} + \frac{I_G}{A\bar{x}} \Rightarrow \bar{h} - \bar{x} = \frac{I_G}{A\bar{x}}$ const, const, increase

$= (\downarrow)$

IAS 05 A vertical rectangular dockgate of 2m wide remain in its position because of water of one in side. The gate weight 800 kg & just sliding down started when the level of water from the bottom of the gate reaching to 4 m is, find μ



$W = \mu F_H$, $\gamma_w = \rho_w \times g$

$mg = \mu(\gamma A \bar{x})$

$800 \times g = \mu (g \times 1000)(4 \times 2)(2)$

$\boxed{\mu = 1/20 = 0.05}$

EM A Horizontal water tank is in the shape of a cylinder with hemispherical end exactly half fully with water and F_x/F_y = (?) on one its hemispherical end

F_y = wt. of liq^n in imaginary vol

$= \gamma_w \times$ vol^m

$= \gamma_w \times \frac{1}{4}\left[\frac{4}{3}\pi r^3\right] = \gamma_w \times \frac{1}{3}\pi r^3$

$F_x = \gamma A \bar{x}$

$= \gamma_w \times \frac{\pi r^2}{2} \times \frac{4R}{3\pi}$

$\therefore \frac{F_x}{F_y} = \frac{\frac{2}{3}\pi r^3}{\frac{1}{3}\pi r^3} = \boxed{\frac{2}{\pi}}$

Horizontal plane

$$\tan\theta = \left(\frac{h_1 - h_2}{L}\right) = \frac{\alpha_x}{g}$$

Interview Pt

C.G will turned

Rear will have dislocation

front wheel are free to move

Non suggested — fully filled — partial filled

Inclined plane

$\alpha_x = \alpha\cos\phi$

$\alpha_y = \alpha\sin\phi$

$$\tan\theta = \frac{\alpha_x}{g \pm \alpha_y}$$

$+ \uparrow$

$- \downarrow$

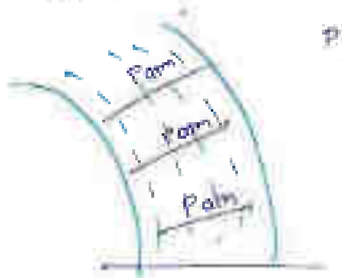
Vertical plane (Motion of lift or Elevator)

$$P = \rho g h\left[1 \pm \frac{\alpha_z}{g}\right]$$

[ESE] NOTE: If the container moving downward with acceleration ↓ $[a_z = g]$ pressure at any point will be zero (atm)

$$p = \rho g h\left[1 \pm \frac{a_z}{g}\right] = \rho g h\left[1 - \frac{g}{g}\right] = 0$$

$P_A = P_B = P_C = P_{atm}$



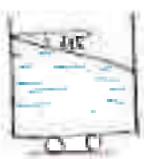
$p = \rho g h\left[1 - \frac{a_z}{g}\right] = 0$ $\{a_z = g\}$

but localize pressure increase

$$z + \frac{P}{\rho g} + \frac{V^2}{2g} = C$$ (const)

The

[ISRO] Acceleration required to cause the free surface of liq^n in container moving on horizontal track to dip by 45° is ?



$$\tan\theta = \frac{a_x}{g} \rightarrow \tan 45^\circ = \frac{a_x}{g}$$

$$\boxed{a_x = g}$$

[IES-07] Open rectangular container with base area $[2 \times 3]$ m² was filled with liq^n $S = 0.8$ to a height of 0.5 m & was imparted an upward acceleration 4.9 m/s² pressure at container at bottom will be ?

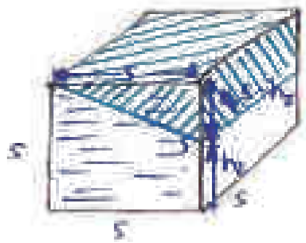
$p = \rho g h\left[1 \pm \frac{a_z}{g}\right]$ $a_z \uparrow$

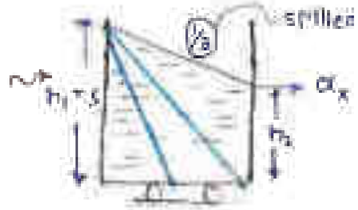
$= 8000 \times 9.81 \times 0.5 \left[1 + \frac{4.9}{9.8}\right]$

$$\boxed{p = 58.8 \text{ kPa}}$$

[ES] A open cubical container completely filled with water is accelerated on a horizontal plane along one of its side. Find the uniform acc^n, such that 1/3 rd of vol^m of water has been spilled out ?

(1)





$$\tan\theta = \frac{h_1 - h_2}{L} = \frac{\alpha_x}{g}$$

$$\alpha_x = \left(\frac{s - h_2}{s}\right) \cdot g$$

$$\text{vol}^m \text{ spilled} = \frac{1}{3}(s^3)$$

$$\frac{1}{2}(s)(s-h_2)(s) = \frac{1}{3}(s^3)$$

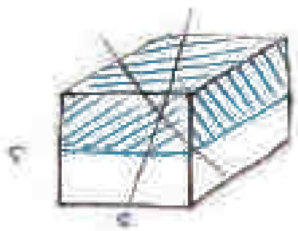
$$s - h_2 = \frac{2}{3}s$$

$$h_2 = \frac{s}{3}$$

$$\tan\theta = \frac{h_1 - h_2}{L} = \frac{s - s/3}{s} = \frac{2s/3}{s} = \frac{\alpha_x}{g}$$

$$\boxed{\alpha_x = \frac{2g}{3}}$$

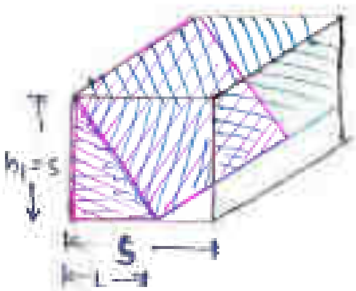
(2) $\frac{1}{2}$ of vol^m spilled out ; $\alpha = ?$



$$\tan 45^\circ = \frac{\alpha_x}{g}$$

$$\boxed{\alpha_x = g}$$

(3) $\frac{2}{3}$ of vol^m spilled out ; $\alpha = ?$



$$\text{vol}^m \text{ spilled} = \frac{2}{3}(s^3)$$

$$\tan\theta = \frac{h_1 - h_2}{L} = \frac{\alpha_x}{g}$$

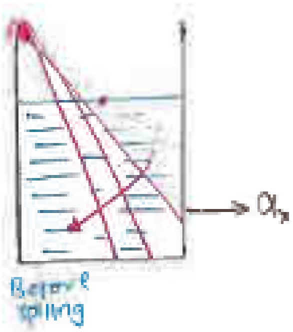
$$= \frac{s - 0}{L} = \frac{\alpha_x}{g}$$

$$\text{Remaining water} = \frac{1}{3}(s^3)$$

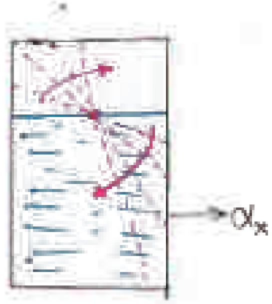
$$\frac{1}{2} \times L \times s \times s = \frac{1}{3}s^3 \Rightarrow L = \frac{2}{3}s$$

$$\alpha_x = \left(\frac{s}{L}\right) g = \left(\frac{s \cdot 3}{2s}\right) g \qquad \boxed{\alpha_x = \frac{3}{2}g}$$

Open



Closed



$$F = Ma_x = (F_{Hyd.\ close})_x \longrightarrow$$

Liquid in Rotation

⇒ Vortex flow

Forced Flow

① Rotation is possible by providing external energy

② $V \propto R$

$\therefore V = R\omega = R\frac{2\pi N}{60}$

$V = \frac{\pi D N}{60}$

③ Rotational flow

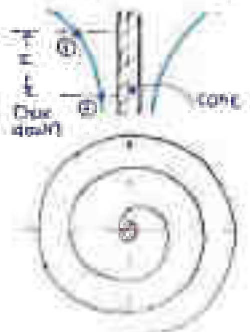
e.g. flow of water in runner of centrifugal pump

↓ within runner ↓ within impeller

Free Flow

① Rotation is possible by conservation of angular momentum

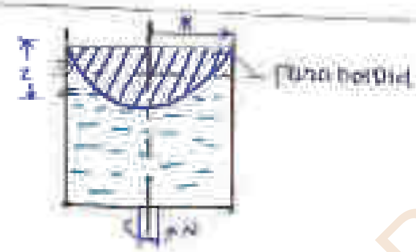
② $V \propto 1/R$; $V = C/R$



③ Irrotational flow

within cone – ER } mixed flow
outside core – R

e.g. flow of water, wash basin whirlpool, flow of water in turbine after leaving runner & in pump after leaving impeller



$Z = \frac{V^2}{2g}$

$$\boxed{Z = \frac{R^2\omega^2}{2g}}$$

⇒ Vol^m spilled = vol^m paraboloid $= \frac{1}{2}\pi R^2 Z$

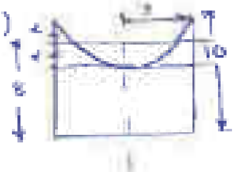
For cylinder  $= \pi R^2 H$

For cone  $= \frac{1}{3}\pi R^2 H$

For paraboloid  $= \frac{1}{2}\pi R^2 H$

Eg 

What will be the maximum speed rpm which water will not spill out N

A) 

$\therefore z = 4$ & $R = 3$

$\therefore z = \dfrac{R^2\omega^2}{2g}$

$\omega^2 = \dfrac{4 \times 2 \times 9.81}{9}$

$\boxed{\omega = 8.72 \text{ rad/s}}$

$N = \dfrac{8.72 \times 60}{2\pi} \Rightarrow \boxed{N = 83.26 \text{ rpm}}$

[1] An open cylinder container of 30 cm diameter & 50 cm height was completely filled with water (liquid) find the amount of liquid spilled out when it is rotating about its axis at (i) 160 rpm (ii) 240 rpm

→ Vol^m of liquid spilled = Vol^m of paraboloid

$= \frac{1}{2}\pi R^2 z$

(i)

but $z = \dfrac{R^2\omega^2}{2g} = \dfrac{(0.15)^2}{2 \times 9.81}\left(\dfrac{2\pi \times 160}{60}\right)^2$

$z = 0.1107$

$V = \frac{1}{2}\pi R^2 z = \frac{1}{2} \times \pi \times (0.15)^2 \times 0.1107$

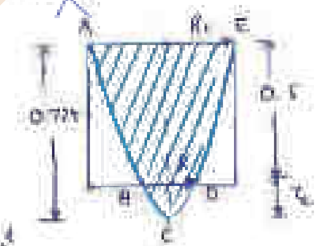
$\boxed{V = 0.01414 \text{ m}^3}$

Amount of spilled = 14.1 litres

(ii) $z = \dfrac{R^2\omega^2}{2g} = \dfrac{(0.15)^2}{2 \times 9.81}\left(\dfrac{2\pi \times 240}{60}\right)^2$

$= 0.7243 = 72.5$ cm $>$ 50 cm (so directly can't find vol^m spilled)

$V = \frac{1}{2}\pi R^2 z = \frac{1}{2} \times \pi \times (0.15)^2 \times 0.7243 = 0.\cancel{0}6$



Vol^m of ABCDE = 0.0255 m^3

→ vol^m of paraboloid BCD,

$V_2 = \frac{1}{2}\pi R_2^2 z_2$

$\therefore z_2 = 0.725 - 0.5 = 0.225$

$z_2 = \dfrac{R_2^2\omega^2}{2g} \Rightarrow R_2^2 = \dfrac{(2 \times 9.81 \times 0.225)(60)^2}{(2\pi \times 240)^2}$

$\therefore R_2 = 0.083$ m

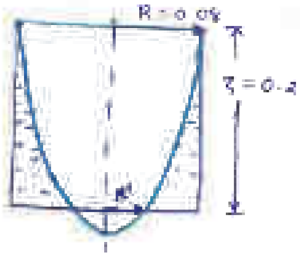
$V_2 = \frac{1}{2}\pi R_2^2 z_2 = \frac{1}{2} \times \pi \times (0.083)^2 \times (0.225) = 0.00244$ m^3

Amount spilled $= V_1 - V_2$

$= 25.5 - 2.4$

$= 23.05$ litres

mm radius was completely filled with water & rotated about its axis. find the uniform speed of rotation such that $\frac{1}{3}^{rd}$ area of base was exposed.



→ $\frac{1}{3}^{rd}$ area of circle exposed

that means

$$\pi R'^2 = \frac{1}{3}\pi R^2$$

$$R' = \frac{R}{\sqrt{3}} = \frac{0.08}{\sqrt{3}} = 0.0461 \text{ m}$$

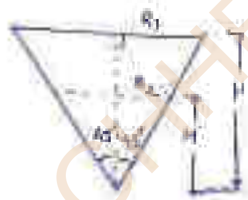
→ $$z = \frac{(R^2 - R'^2)\,\omega^2}{2g}$$

$$\omega^2 = \frac{0.2 \times 2 \times 9.81}{(0.08)^2 - (0.0461)^2} = 917.93 = \frac{2\pi N}{60}$$

$$\boxed{N = 289.34 \text{ rpm}}$$

IAS 3) A Right circular conical container with its apex downward & axis vertical was exactly half full. Find the speed of rotation about its axis when the water is about to spill for radi of 1.5 m

$$z = \frac{R^2\omega^2}{2g}$$



→ vol^m of paraboloid $= \frac{1}{2}$ (vol^m of cone)

$$\frac{1}{2}\pi R^2 z = \frac{1}{2} \times \frac{1}{3}\pi R^2 H$$

$$\boxed{z = H/3} = R/3 \quad \boxed{\theta = 45^\circ}$$

$$\tan 45^\circ = R/H$$

→ $$z = \frac{R^2\omega^2}{2g}$$

$$\omega^2 = \frac{2g \times R/3}{R^2} = \frac{2g}{3R} = \frac{2 \times 9.81}{3 \times 1.5} = 4.36$$

$$\frac{2\pi N}{60} = \omega = 2.08$$

$$N = \frac{2.08 \times 60}{2\pi}$$

$$\boxed{N = 19.93 \text{ rpm}} \text{ Ans}$$

⇒ Fundamental of fluid flow :

① Steady & unsteady flow

- If all the property doesn't vary w.r.t time then the flow is said to be steady flow
  If single property changes with respect to time it will be considered as unsteady flow

- property = f (space , time) = f (x, y, z, t)

→ $\left.\frac{\partial P}{\partial t} = 0\right)_{all}$ ⇒ steady (All the property w.r.t time zero

→ $\frac{\partial P}{\partial t} \neq 0$ ⇒ unsteady (even single property varies)

② Uniform & Non uniform flow

→ $\frac{\partial P}{\partial s} = 0$ ⇒ uniform (i.e. $\frac{\partial P}{\partial x}, \frac{\partial P}{\partial y}, \frac{\partial P}{\partial z} = 0$)

→ $\frac{\partial P}{\partial s} \neq 0$ ⇒ Non uniform

Eg

$Q = A_1 V_1$
As area increase , velocity decrease & vice v
$\frac{\partial P}{\partial s} \neq 0$
⇒ non uniform flow.

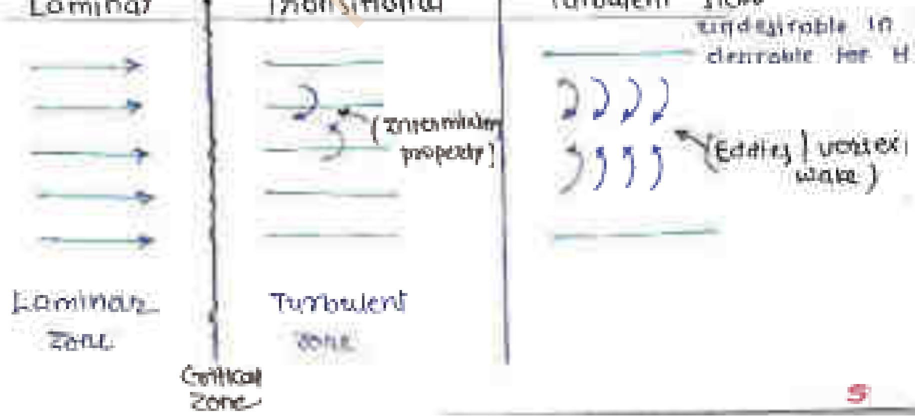
but we can't give the answer about steady & unsteady because to find all property need to be checked.

③ Compressible & Incompressible flow

- Gas is compressible & liqn is incompressible → ρ = const
- Gas can give incompressible when $M < 0.3$

④ Laminar | Transitional | Turbulent flow

undesirable in F
desirable for H.T



- If the fluid particles rotate about their masses center while moving forward, then the flow is said to be rotational otherwise irrotational.

Note: (1) Rotational is because of variation in shear i.e. difference in velocity of adjacent layer

→ if, $\omega_x, \omega_y, \omega_z = 0$ then irrotational
$\omega_x, \omega_y, \omega_z \neq 0$ the rotational

➡ Velocity & Acceleration :

$v = f\,[x, y, z, t]$

Velocity = $\frac{\text{dist}^n}{\text{time}}$

Accle. = $\frac{\Delta \text{velocity}}{\text{time}}$

Jerk = $\frac{\Delta \text{Accl}^n}{\text{time}}$

$$v \begin{cases} x \rightarrow V_x = u \longrightarrow a_x \\ y \rightarrow V_y = v \longrightarrow a_y \\ z \rightarrow V_z = \omega \longrightarrow a_z \end{cases}$$

so, $\vec{V} = V_x \hat{i} + V_y \hat{j} + V_z \hat{k}$

$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

so, $V_x = \frac{dx}{dt} = u$; $V_y = \frac{dy}{dt} = v$; $V_z = \frac{dz}{dt} = \omega$

we can't directly put $u = \frac{dx}{dt}$ bcoz one particle move ~~from~~ one corner to another is rotating x, y, z, & t

$a_x = \frac{du}{dt}$; $a_y = \frac{dv}{dt}$; $a_z = \frac{d\omega}{dt}$

so, $u = f[x, y, z, t]$
$v = f[x, y, z, t]$
$z = f[x, y, z, t]$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\cdot\underbrace{\left(\frac{dx}{dt}\right)}_{u} + \frac{\partial u}{\partial y}\cdot\underbrace{\left(\frac{dy}{dt}\right)}_{v} + \frac{\partial u}{\partial z}\underbrace{\left(\frac{\partial z}{\partial t}\right)}_{\omega} + \frac{\partial u}{\partial t}$$

IMP.

$$\boxed{a_x = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \omega\frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}}$$

$$\boxed{a_y = u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + \omega\frac{\partial v}{\partial z} + \frac{\partial v}{\partial t}}$$

$$\boxed{a_z = u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} + \omega\frac{\partial \omega}{\partial z} + \frac{\partial \omega}{\partial t}}$$

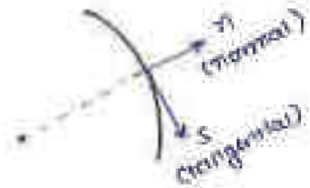
time

$V = \sqrt{u^2+v^2+w^2}$ →

$$\boxed{a_{total} = \underset{\{x,y,z\}}{a_{convective}} + \underset{\{t\}}{a_{local\ (temporal)}}}$$

for static fluid $a_{local} = 0$

→ for - 2-D



$V = f[s, n, t]$

$V \begin{cases} s \longrightarrow V_s \longrightarrow a_s \\ n \longrightarrow V_n \longrightarrow a_n \end{cases}$

so, $V_s = \frac{ds}{dt}$; $V_n = \frac{dn}{dt}$

$a_s = \frac{dV_s}{dt}$; $a_n = \frac{dV_n}{dt}$

also $\underline{V_s, V_n} = f[s, n, t]$

$$a_s = \frac{\partial V_s}{\partial s}\left(\frac{ds}{dt}\right) + \frac{\partial V_s}{\partial n}\left[\frac{dn}{dt}\right] + \frac{\partial V_s}{\partial t}$$

$$a_s = V_s \cdot \frac{\partial V_s}{\partial s} + V_n \frac{\partial V_s}{\partial n} + \frac{\partial V_s}{\partial t}$$

& $$a_n = \underbrace{V_s \frac{\partial V_n}{\partial s} + V_n \frac{\partial V_n}{\partial n}} + \frac{\partial V_n}{\partial t}$$

$$a_t = a_{convective} + a_{local}$$

→ Continuity Equation

→ Conservation of mass →

m = mass flow rate is constant

$\dot{m} = \frac{mass}{time}$ = const ⇒ $\frac{kg}{s}$ = const

$$\dot{m} = \frac{kg}{s} \times \frac{m^3}{m^3} = \underset{\rho}{\frac{kg}{m^3}} \cdot \underset{Q}{\frac{m^3}{sec}} = \underset{\rho}{\frac{kg}{m^3}} \cdot \underset{A}{m^2} \cdot \underset{V}{\frac{m}{sec}} = const$$

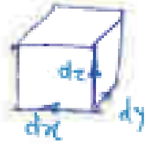
$$\dot{m} = \rho Q = \rho A V = const$$

→ For Incompressible flow $\underline{\rho \text{ is const}}$

$$\boxed{A_1V_1 = A_2V_2}$$

Gate 9"

$$\boxed{\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0}$$ conservation of mass



→ for steady flow $\boxed{\frac{\partial \rho}{\partial t} = 0}$ (The flow doesn't change w.r.t time)

→ for Incompressible flow ⇒ $\rho = \text{const}$

$$\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0}$$

⇒ For 2-D flow :

$\boxed{\nabla \cdot \vec{V} = 0}$ ⇒ $\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$ (Incompressible 2-D flow)

* $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$ ⇝ incompressible ($\rho = \text{const}$)

* $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} \neq 0$ ⇝ compressible

GATE 2003 $\vec{V} = 2y\hat{i} + 3x\hat{j}$ the convective accⁿ in x direction at point (1,1) = (?)

→ $\vec{V} = 2y\hat{i} + 3x\hat{j}$

$V_x = 2y = \frac{dx}{dt}$ $V_y = 3x = \frac{dy}{dt}$

$$a_x = \frac{dV_x}{dt} = \frac{\partial V_x}{\partial x}\left(\frac{\partial x}{\partial t}\right) + \frac{\partial V_x}{\partial y}\left(\frac{\partial y}{\partial t}\right)$$

$= (0)(2y) + (2)(3x)$

$= 6x$ @ (1,1)

$\boxed{a_x = 6}$ convective

Ex For 2-D incompressible flow x-component of velocity $u = c \ln(xy)$ the $v =$ (?)

$\frac{\partial u}{\partial x} = c \cdot \frac{1}{xy} \cdot (1)y = \frac{c}{x}$

$\frac{\partial v}{\partial y} = -\frac{c}{x}$ but for I.C. $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$dv = -\frac{c}{x} \int dy$

$$\boxed{v = -c\left(\frac{y}{x}\right)}$$

ES Incompressible

$u = \lambda x y^3 - x^2 y$

$v = xy^2 - \frac{3}{4} y^4$

What value of λ = (?)

→ $\frac{\partial u}{\partial x} = \lambda(1)(y^3) - 2xy$ & $\frac{\partial v}{\partial y} = 2xy - \frac{3}{4}(4y^3)$

$= 2xy - 3y^3$

$\lambda y^3 - 2xy + 2xy - 3y^3 = 0$ for incompressible $\boxed{\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0}$

$\boxed{\lambda = 3}$

ES-06 $\vec{V} = (5x + 6y + 7z)\hat{i} + (3x + 5y + 4z)\hat{j} + (2x + 3y + \lambda z)\hat{k}$

where λ = const = (?)

inorder that mass is conserved. (compressible) $\boxed{\text{comp \& unsteady}}$

density varies $\rho = \rho_0 e^{-2t}$

→ $\rho = \rho_0 e^{-2t}$ ⇒ $\frac{\partial \rho}{\partial t} = \rho_0 e^{-2t}(-2) = -2\rho_0 e^{-2t} = -2\rho$

→ $\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) + \frac{\partial \rho}{\partial t} = 0$

$\rho \frac{\partial}{\partial x}(5x + 6y + 7z) + \rho \frac{\partial}{\partial y}(3x + 5y + 4z) + \rho \frac{\partial}{\partial z}(2x + 3y + \lambda z)$

$+ (-2\rho) = 0$

$\rho(5) + \rho(5) + \rho(\lambda) = 2\rho$

$\boxed{\lambda = -8}$

→ Here density varies so, $\rho = \rho_0 e^{-2t}$ so; compressible

→ Even single property change with time then unsteady

GATE-08/15 "The continuity eq^n $\nabla \cdot \vec{V} = 0$ to be valid which of following necessary cond^n

(i) steady

(ii) unsteady

(iii) comp

(iv) incomp

→ $\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} + \frac{\partial \rho}{\partial t} = 0$

for steady $\frac{\partial}{\partial t}[\;] = 0$ → $\frac{\partial \rho}{\partial t} = 0$

so will not $\nabla \cdot \vec{V} \neq 0$

→ $\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}$

For incomp ; ρ = const.

$\rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right] + \frac{\partial}{\partial t}[\text{const}] = 0$

$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$

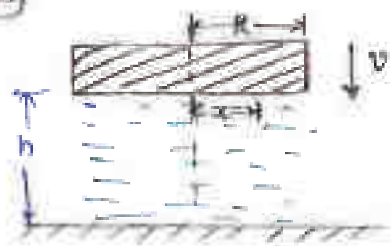
- If fluid is incompressible then it may be steady / unsteady.
- If fluid steady then it must be compressible.

So fluid is said to be ~~...~~ $\boxed{\text{steady}}$ then it will be ~~compressible~~ $\boxed{\text{incompressible}}$

→ but if fluid is ~~compressible~~ $\boxed{\text{incompressible}}$ then it must be steady (or) unsteady both

– i.e. ρ = const but we can not say w.r.t. time it is not change directly so it may be steady or unsteady

GATE-03



The gap betn a moving circular plate and stationary surface is being continuously reduced, as circular plate coming down with uniform velocity V as shown; Assuming the fluid in between is incompressible and its way flowing out radially.

① The radial velocity V_r at any radius 'r', when gap width is 'h' (?)

(a) $\frac{Vr}{h}$ (b) $\frac{Vr}{2h}$ (c) $\frac{2Vh}{R}$ (d) $\frac{Vh}{R}$

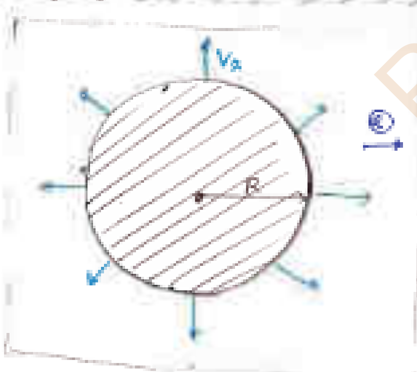
② Radial component of acceleration at $r = R$

(a) $\frac{V^2R}{2h^2}$ (b) $\frac{V^2R}{4h^2}$ (c) $\frac{3V^2R}{2h^2}$ (d) $\frac{3V^2R}{4h^2}$

(h = c) (h = var)

→

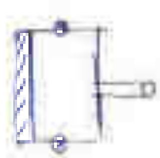
	h ① ↓	r ② →
Velocity	V	V_r
Area	πr^2	$2\pi r h$



① $A_1V_1 = A_2V_2$

$\pi r^2 V = V_r(2\pi r h)$

$$\boxed{V_r = \frac{Vr}{2h}}$$



(2) $V_x = \frac{dx}{dt} = \frac{Vx}{dt}$; $V_h = \frac{dh}{dt}$

- $h =$ const ;

$$a_x = \frac{dV_x}{dt} = \frac{d}{dt}\left[\frac{Vx}{2h}\right] = \frac{V}{2h}\left[\frac{dx}{dt}\right]$$

$$= \frac{V}{2h} \cdot V_x = \frac{V}{2h}\left[\frac{Vx}{2h}\right]$$

$$\boxed{a_x = \frac{V^2 x}{4h}} \rightarrow \text{at } x = R$$

$$\boxed{a_R = \frac{V^2 R}{4h}}$$

- $h =$ variable :

$$a_x = \frac{dV_x}{dt} = \frac{d}{dt}\left[\frac{Vx}{2h}\right] =$$

$V = f[x, h, t]$

$V \rightarrow x \rightarrow V_x \rightarrow a_x$

$V \rightarrow h \rightarrow V_h \rightarrow a_h$

$\therefore V_x = \frac{dx}{dt} = \frac{Vx}{2h}$ & $V_h = \frac{dh}{dt} = -V$

Height is decreases so take value -ve, change of time appre but height is decrease in nature.

$a_x = \frac{dV_x}{dt}$

so, $a_x = \frac{d}{dt}[V_x] = \frac{\partial V_x}{\partial x}\left(\frac{\partial x}{\partial t}\right) + \left(\frac{\partial V_x}{\partial h}\right)\left(\frac{\partial h}{\partial t}\right) + \frac{\partial V_x}{\partial t}$ ($f(x, h, t)$)

$$= \frac{\partial}{\partial x}\left[\frac{Vx}{2h}\right] \times \left[\frac{Vx}{2h}\right] + \frac{\partial}{\partial h}\left[\frac{Vx}{2h}\right] \cdot [-V] + \frac{\partial}{\partial t}\left[\frac{Vx}{2h}\right]$$

$$= \frac{V}{2h}\left[\frac{Vx}{2h}\right] + \frac{Vx}{2}\left(\frac{-1}{h^2}\right)(-V) + 0$$

$$= \frac{V^2 x}{4h} + \frac{V^2 x}{2h^2}\frac{(2)}{(2)}$$

$$\boxed{a_x = \frac{3V^2 x}{4h^2}}$$

Q.5) For a 2-D ; I.C. flow ; x-component of velocity is $U = Ae^x$ then $V = (?)$

$\rightarrow \frac{du}{dx} + \frac{dv}{dy} = 0$

$\frac{d}{dx}(Ae^x) = -\frac{dv}{dy}$

$-Ae^x \int dy = \int dV$

$\boxed{V = -Ae^x y + f(x)}$

Here, we use $f(x)$ const. being doing integration w.r.t. y so x const.

- line → direction
- function → Magnitude

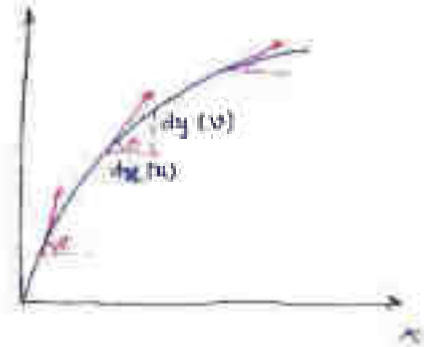
[Tracing motion of different particle]

- It is imaginary curve drawn in flow field, such that the tangent to it at any point will give the direction of velocity at that point.

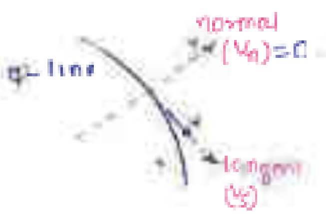
→ $\boxed{\tan\theta = \frac{dy}{dx} = \frac{v}{u} = m_\psi}$ slope of stream line

⇒ $\boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$ ← for 3-D

$v\,dx - u\,dy = C$



NOTE :- The flow across a stream line will be zero



$V_x = V\cos\theta$
$V_y = V\sin\theta$

$V_x = V$
$V_y = 0$

Note :- Potential line [ϕ - line] will be orthogonal [⊥] to stream line

$m_\phi = \frac{-1}{m_\psi} = \left(-\frac{u}{v}\right)$ & $m_\psi = +\frac{v}{u}$

→ line passing from $[x_1, y_1]$ with slope m is
$[y - y_1] = m(x - x_1)$

→ Path line - / [Lagrangian Approach] (Stream line)
It is line traced by a single particle over a period of time

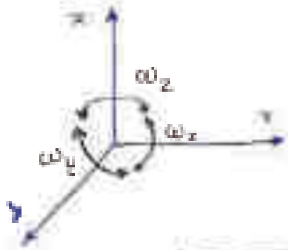
[Tracing of any one fluid particle]

→ Streak line
(or) filament line

[Eulerian Approach]

[Identification of location of number of fluid particles] cigarette smoke

Rotation



Rotational velocity vector

$\vec{\omega} = \omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}$

For 2-D:

$2D_{xy} \longrightarrow \omega_z = 0 \longrightarrow$ Irrotational (No Rotation)

$\omega_z \neq 0 \longrightarrow$ Rotational

u	v	ω
x	y	z
y	z	x
z	x	y

$$\boxed{\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]}$$

$$\boxed{\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]}$$

$$\boxed{\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]}$$

$$\rightarrow \omega = \frac{1}{2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & \omega \end{vmatrix}$$

Circulation

$$\boxed{\text{Circulation}_{(xy\text{-plane})} = 2 \times \omega_z \times \text{Area}}$$

$\text{Circulation}_{(xy)} = 2 \times \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \cdot \text{area}$ (Regular geometry)

$\text{Circulation}_{(xy)} = \int_y \int_x \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] \cdot \text{area}$ (Irregular geometry)

Vorticity (Ω or ξ)

$$\boxed{\text{Vorticity}_{(xy\text{-plane})} = \frac{\text{Circulation}}{\text{Area}}}$$

Ω -

$$\boxed{\text{Vorticity}_{xy} = 2\omega_z}$$

So vorticity vector

$$\boxed{\text{Vorticity vector} = 2\left[\omega_x \hat{i} + \omega_y \hat{j} + \omega_z \hat{k}\right]}$$

Q.08) (1, 2) = (?)

a) $x - 2y = 0$ b) $y - 2x = 0$

c) $x + 2y = 0$ d) $y + 2x = C$

$\tan\theta = \frac{dy}{dx} = \frac{v}{u}$

$\vec{V} = \underline{ax}\,\hat{i} + \underline{ay}\,\hat{j}$

$u = ax$

$v = ay$

$\frac{dy}{dx} = \frac{ay}{ax}$

$x\,dy = dx\,y$

$\int \frac{dy}{y} = \int \frac{dx}{x}$

$\ln y = \ln x + \ln c$

$y = xc$ at (1, 2) passing

$2 = 1(1)c \rightarrow \boxed{c = 2}$

$y = 2x \Rightarrow \boxed{y - 2x = 0}$

For 2-D flow x & y given then

$\tan\theta = \frac{dy}{dx} \rightarrow \boxed{\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}}$ put in that form

$= \frac{w}{u}$ 3D

GATE-01) fluid is incompressible & irrotational

Q-09)

P) $u = 2x$; $v = 3y$

Q) $u = 2xy$; $v = 0$

R) $u = 2x$; $v = -2y$

(a) P & R (b) Q & R

(c) Q (d) R

for comp (or) incompress	for Rotation (or) irrotation
$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$	$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

P) $\rightarrow \frac{\partial}{\partial x}(2x) + \frac{\partial}{\partial y}(3y) = 2 + 3 = 5 \neq 0$ P ✗

R) $\rightarrow \frac{\partial}{\partial x}(2x) + \frac{\partial}{\partial y}(-2y) = 2 - 2 = 0$, $\omega_z = \frac{1}{2}(0 - 0) = 0$

ES-09) $V = 3xy^2 i + 2xy j + (2zy + 3t) k$

Rotational velocity vector at [1, 2, 1] at t = 5

$u = 3xy^2, \quad v = 2xy, \quad w = 2zy + 3t$

u	v	w
x	y	z
y	z	x
z	x	y

$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right] = \frac{1}{2}(2z$

$\vec{\omega} = \omega_x \bar{i} + \omega_y \bar{j} + \omega_z \bar{k}$

$i - 4k$

$$= \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial y \\ 3xy^2 & 2xy & 2zy + 3t \end{vmatrix}$$

$= i(2z +$

GATE -10) $\vec{v} = 2xy\bar{i} - x^2 z \bar{j}$ vorticity vector [1, 1, 1]

vorticity vector $= 2[\omega_x \bar{i} + \omega_y \bar{j} + \omega_z \bar{k}]$

$u = 2xy$

$v = -x^2 z$

$w = 0$

u	v	w
x	y	z
y	z	x
z	x	y

$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$

$= \frac{1}{2}[0 - (-x^2)] = \frac{x^2}{2} = \frac{1}{2}$

$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$

$= \frac{1}{2}[0 - 0] = 0$

$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$

$= \frac{1}{2}[-2xz - 2x] = -(x + xz) = -2$

$\omega = \omega_x \bar{i} + \omega_y \bar{j} + \omega_z \bar{k}$

$= \frac{x^2}{2}\bar{i} + 0 + (-(x + xz))\bar{k}$

$= \frac{1}{2}[-1 - 1] = \boxed{1/2}$

velocity vector $= 2[\omega_x \bar{i} + \omega_y \bar{j} + \omega_z \bar{k}]$

$= 2\left[\frac{1}{2}\bar{i} + 0\bar{j} + (-2)\bar{k}\right]$

$= \underline{i - 4k}$

GATE-14) $V = K|y| - xK|$

$u = y$, $v = 0$, $w = -x$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \frac{1}{2}[0-1] = -\frac{1}{2}K$$

$$\text{vorticity}_{(xy)} = 2\omega_z = -K$$

$$\Omega_z = 2\omega_z = 2\left(\frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]\right)$$

GATE 01) A circulation around a circle of radius 2 unit for the flow field given by $u = 2x + 3y$; $v = -2y$ (xy-plane)

$$\rightarrow \omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = \frac{1}{2}[0-3] = -\frac{3}{2}$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial u}{\partial y} - \frac{\partial}{\partial x}\right]$$

(I) circulation (xy) $= 2\omega_z(\text{area}) = 2\left(-\frac{3}{2}\right)(\pi(2)^2)$

$= \underline{-12\pi}$ units

GATE 01) For a flow through nozzle, incompressible flow the flow velocity along the nozzle axis
$V = u_0\left[1 + \frac{3x}{L}\right]\hat{i}$ where L = length of the nozzle & x is distance from its inlet end, find the time req^n to for a fluid particle on the nozzle axis to travel from from its inlet plane to exit plane.

$$\rightarrow V = \underbrace{u_0\left[1 + \frac{3x}{L}\right]}_{u}\hat{i}$$

$$u = u_0\left[1 + \frac{3x}{L}\right]$$

Here x is vary increases with v so, V↑ nozzle is convergent

$$\text{time} = \frac{\text{dist}}{\text{velocity}}$$

for travel dx dist^n time req^n & integrate to find whole tim

$$t = \int dt = \int_0^L \frac{dx}{u_0\left[1 + \frac{3x}{L}\right]} = \frac{1}{u_0}\frac{\left\{\ln\left[1 + \frac{3x}{L}\right]\right\}_0^L}{\frac{3}{L}}$$

$$= \frac{L}{3u_0}\left[\ln\left[1 + \frac{3L}{L}\right] - \ln[1+0]\right]$$

$$\boxed{t = \frac{L}{3u_0}\ln 4}$$

Second method

$$V = u_1 = u_0\left[1 + \frac{3x}{L}\right] t$$

$$u = u_0\left[1 + \frac{3x}{L}\right] = \frac{dx}{dt}$$

$$t = dt = \int_0^L \frac{dx}{u} = \int_0^L \frac{dx}{u_0\left[1 + \frac{3x}{L}\right]}$$

Potentional function : (ϕ - function)

- It is function of space & time defined such that negat[ive] derivative w.r.t. any direction will give the component of velocity in that direction.

— $\phi = f\,[\text{space}, \text{time}]$

$$-\left[\frac{\partial \phi}{\partial x}\right] = u$$

$$-\left[\frac{\partial \phi}{\partial y}\right] = v$$

$$-\left[\frac{\partial \phi}{\partial z}\right] = \omega$$

ϕ - function is for 3-D

- A line which ϕ = const then it is equipotential line

NOTE: Negative sign indicates flow will always in direction o[f] decreasing potentional.

$\phi_1 > \phi_2$

$$\text{so,}\quad \frac{\partial \phi}{\partial x} = \frac{\phi_2 - \phi_1}{x_2 - x_1} = \frac{(-ve)}{L}$$

NOTE 2: If ϕ function is continous then it must be irrotation

$$\text{I.R.} = \omega_z = 0$$

$$\Rightarrow \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0$$

$$\Rightarrow \frac{1}{2}\left[\frac{\partial}{\partial x}\left[\frac{-\partial \phi}{\partial y}\right] - \frac{\partial}{\partial y}\left[\frac{-\partial \phi}{\partial x}\right]\right] = 0$$

$$\Rightarrow -\frac{\partial^2 \phi}{\partial x\, \partial y} + \frac{\partial^2 \phi}{\partial y\, \partial x} = 0$$

$$\boxed{\frac{\partial^2 \phi}{\partial x\, \partial y} = \frac{\partial^2 \phi}{\partial y\, \partial x}} \Rightarrow \text{Continous function}$$

→ If ϕ function existing then it will be irrotation

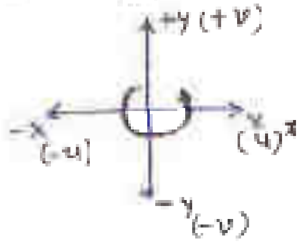
ψ - function : (Stream function)

It is function of space & time define such that the derivative w.r.t. to any direction will give the component of velocity at right angle in clockwise direction

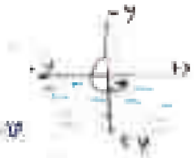
$$\boxed{\frac{\partial \psi}{\partial x} = v}$$

$$\boxed{V = \sqrt{v^2 + u^2}}$$

$$\boxed{\frac{\partial \psi}{\partial y} = -u}$$



- ψ - function is valid for 2-D function



If case 2

$\frac{\partial \psi}{\partial x} = -v$

& $\frac{\partial \psi}{\partial y} = -u$

Note :- If ψ f^n satisfy Laplace equation it must be irrotational.

$\omega_z = 0$ (for irrotational)

$$\therefore \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0 \qquad \text{put} \quad v = \frac{\partial \psi}{\partial x} \;\&\; u = -\frac{\partial \psi}{\partial y}$$

$$\therefore \frac{1}{2}\left[\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) - \frac{\partial}{\partial y}\left(-\frac{\partial \psi}{\partial y}\right)\right] = 0$$

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0}$$

so $\boxed{\nabla^2 \psi = 0}$

Cauchy - Remain's Eq^n → (CR eq^n)

$$u = -\frac{\partial \phi}{\partial x} = -\frac{\partial \psi}{\partial y}$$

$$\Rightarrow \boxed{\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}}$$

$$v = -\frac{\partial \phi}{\partial y} = \frac{\partial \psi}{\partial x}$$

$$\boxed{\frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}}$$

GATE CIVIL $\psi = 3xy$ then find velocity $(2,3) = ?$

$$u = \frac{\partial \psi}{\partial x} = \frac{\partial (3xy)}{\partial x} = 3y$$

$$v = -\frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(3xy) = -3x$$

$$V = \sqrt{u^2 + v^2} = \sqrt{(3y)^2 + (-3x)^2}$$
$$= \sqrt{9(9) + 9(4)} = \sqrt{117}$$
$$= \underline{10.8} \ m/s$$

GATE (2003) $\phi = \log_e (x^2 + y^2)$ then $\psi = ?$

$$\frac{\partial \phi}{\partial x} = \frac{1}{(x^2+y^2)}(2x) \quad \& \quad \frac{\partial \phi}{\partial y} = \frac{1}{(x^2+y^2)}(2y)$$

$$\frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y} \Rightarrow \frac{2x}{(x^2+y^2)} = \frac{\partial \psi}{\partial y}$$

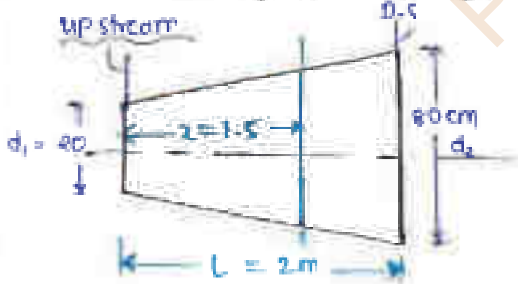
$$\frac{1}{1 + y^2/x^2}$$

$$\int \partial \psi = \int \frac{2x}{(x^2+y^2)} \partial y$$
$$= 2 \int \frac{x}{x^2[1 + (y/x)^2]} \partial y$$
$$= 2 \int \left(\frac{1/x}{1 + (y/x)^2} \right) dy$$

$$\int \frac{1/x}{1 + (y/x)^2} = \tan^{-1} a$$

$$\boxed{\psi = 2 \tan^{-1}(y/x) + f(x)}$$

ES-2000) A two meter long conical diffuser with 20 cm dia. upstream end, at 80 cm dia. at downstream end at an end instant flow rate was estimated 200 lit/s & it was found to increase at a rate of 50 lit/s estimate local, convective, & total acceleration at a dist^n of 1.5 m from upstream end.



$$a_x = u\left(\frac{\partial u}{\partial x}\right) + \frac{\partial u}{\partial t}$$
$$\quad\ \ \text{conv} \qquad \text{local}$$

$$u_x = \frac{Q}{A_x} = \frac{Q}{\frac{\pi}{4}[d_x]^2} \Rightarrow (1)$$

$$d_x = d_1 + \left(\frac{d_2 - d_1}{L}\right) x = 0.2 + \left(\frac{0.8 - 0.2}{2}\right) x$$
$$= [0.2 + 0.3x]$$

$$\text{so } u_x = \frac{Q}{\frac{\pi}{4}[0.2 + 0.3x]^2}$$

at an instant 2 sec P.S → $Q = 200 \times 10^{-3} = 0.2 \ m^3/s$

increasing 50 L/s → $\frac{\partial Q}{\partial t} = 0.05 \ m^3/s$

→ **Local α**

$$\alpha_{local} = \frac{\partial u}{\partial t}$$

$$= \frac{\partial}{\partial t}\left[\frac{Q}{A_x}\right] \quad \{ \text{But area at } x \text{ dist}^n \text{ is const} \}$$

$$= \frac{1}{A_x}\left[\frac{\partial Q}{\partial t}\right] = \frac{4}{\pi[0.2+0.3\times1.5]^2} \times [0.05]$$

$$\boxed{\alpha_{local} = 0.15 \ m/s^2}$$

→ **Convective**

$$\alpha_{conv} = u\left[\frac{\partial u}{\partial x}\right]$$

so, but $u = \frac{Q}{\pi/4[0.2+0.3x]^2}$

$$\frac{\partial u}{\partial x} = \frac{\partial}{\partial x}\left[\frac{4Q}{\pi[0.2+0.3x]^2}\right]$$

$$= \frac{4Q}{\pi}\frac{\partial}{\partial x}\{(0.2+0.3x)^{-2}\}$$

$$= \frac{4}{\pi}Q(-2)(0.2+0.3x)^{-3}(0+0.3)$$

$$= -0.55$$

$$u_x = \frac{Q}{A_x} = \frac{Q}{\pi/4[0.2+0.3x]^2} = \frac{0.2}{\pi/4[0.2+0.3\times1.5]^2}$$

$$= 0.6$$

$$\alpha_{conv} = (0.6)(-0.55)$$

$$\boxed{\alpha_{conv} = -0.33 \ m/s^2}$$

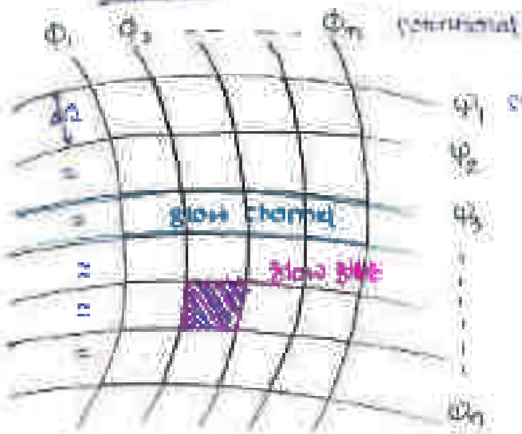
$$\alpha_{total} = \alpha_{conv} + \alpha_{local}$$

$$= -0.33 + 0.15$$

$$\boxed{\alpha_{total} = -0.18 \ m/s^2}$$

→ Flownet Analysis

- Flownet : Flownet is a graphical representation of solution Laplace equation



Property : ① It consist of ϕ line & ψ line which are perpendic to eachother → at all point expect stagnation point.

② The enclosed area are Approximated square.

③ To measure flow rate

GATE Imp → Let ΔQ, is the flow rate through each flow channel; which remains const & is equal to all all the channel.

at eddies formation ΔQ rate is not constant

$$\boxed{\Delta Q = \psi_1 \sim \psi_2 = \psi_2 \sim \psi_3}$$

$$\boxed{Q = n_f \times \Delta Q}$$

Where : n_f = no. of flow channels
= (No. of ψ line − 1)

GATE-07 (3) $\psi = \frac{3}{2}[y^2 - x^2]$ flow rate across the line joining $A[0,3]$ & $B[3,4]$
(x_1, y_1) (x_2, y_2)

$\Delta Q = \psi_1 - \psi_2$

$= \frac{3}{2}[(y_1^2 - x_1^2) - (y_2^2 - x_2^2)] = \frac{3}{2}[\{(3)^2 - (0)^2\} - \{(4)^2 - (3)^2\}]$

$= \frac{3}{2}[9 - 16 + 9] = 3$

Fluid - Dynamics

(Flow application / force & pressure - Dynamics)

Bernoulli's Equation

Conservation of energy

$$\boxed{z + \frac{p}{\rho g} + \frac{v^2}{2g} = \text{const}}$$

where : z = Datum (or) potential Head

$\frac{p}{\rho g}$ = pressure energy Head

$\frac{v^2}{2g}$ = velocity (or) kinetic energy Head ; v = mean velocity

$\left(z + \frac{p}{\rho g}\right)$ = Piezometric (or) static pressure Head

PSU → Bernoulli's eq^n is energy const then why liq^n flowing

- so actually flow from high gradient to low gradient fluid flowing having some losses in it so $E_1 = E_2 + h_{loss}$ but ideal fluid case neglect so we say energy is const but we can't say its direction.

GPVTSC

- ✓ F_G = Gravity force
- ✓ F_p = pressure force
- ✓ F_v = viscous force
- ✓ F_T = turbulent force
- ✓ F_S = surface tension force
- ✓ F_C = compressibility force

① $F = ma_x = F_G + F_p + F_v + F_T + F_S + F_C$ → Newton's equation

neglect F_S, F_C

② $F = ma_x = F_G + F_p + F_v + F_T$ → Reynold's eq^n

neglect F_T

③ $F = ma_x = F_G + F_p + F_v$ → Navier's stock eq^n

neglect F_v

④ $F = ma_x = F_G + F_p$ → Euler's eq^n — Bernoulli's eq^n

from gravity & pressure force Bernoulli's eq^n is derived.

→ By integrating Euler's equation → Bernoulli's eq^n is achieved

→ Bernoulli's eq^n used for ideal fluid

① $z + \frac{P}{\rho g} + \frac{V^2}{2g} = \text{const}$ → Energy head / per unit weight

which of fluid is
unit :- m of flowing fluid

② $gz + \int \frac{dP}{\rho} + \frac{V^2}{2} = \text{const}$ → Per unit mass

③  → Energy conserved in ideal fluid → $E_1 = E_2$

④ Energy Equation for Real fluid flow :- [Eqⁿ → i.e.]

$$\boxed{\left[z_1 + \frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g}\right] = z_2 + \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + h_{loss}}$$

$E_1 = E_2 + h_{loss}$

where = α = K.E. correction factor
$= KE_{act} / KE_{mean}$

→ For Bernoulli's eqⁿ V use is mean velocity.

GATE CIVIL

NOTE :-

Usage
→ coefficient
→ momentum

$V_{max} = V_0$

V_{act} —— ⇒ $Q = \int dQ = \int V_{act} \, dA$

$V_{min} = 0$

$V_{max} = V_0$ = free stream velocity

$V_{mean} = \cancel{\frac{(V_{max} + V_{min})}{2}}$ (In Example V given which is Vmean)

$$\boxed{V_{mean} = \frac{1}{A} \int V \, dA}$$

α = K.E correction factor

$= \frac{(K.E)_{act}}{(KE)_{mean}} = \frac{\frac{1}{2} \rho \int V_{act}^3 \, dA}{\frac{1}{2} \rho A V_{mean}^3}$

$$\boxed{\alpha = \frac{(K.E)_{act}}{(K.E)_{mean}} = \frac{\int V_{act}^3 \, dA}{V_{mean}^3 A}} = 2.0 \left\{ \text{laminar flow through circular pipe} \right\}$$

$\boxed{\alpha = 2.0}$ for laminar flow pipe

→ β = momentum correction factor

$$\boxed{\beta = \left[\frac{\text{actual momentum}}{\text{mean momentum}}\right] = \frac{\int V_{act}^2 \, dA}{V_{mean}^2 A}}$$

GATE 07 | At two points ① & ②, where the velocity are V & 2V, in a Horizontal pipe line; neglecting the losses the pressure drop between the two points

→ $\frac{P_1}{\rho g} + \frac{V^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{4V^2}{2g} + z_2$

$P_1 - P_2 = (2V^2 - \frac{V^2}{2}) \rho g$

$\boxed{P_1 - P_2 = 1.5 \rho V^2}$

Hints : ① Horizontal pipe line ;
Both the points are at same elevation
$\boxed{z_1 = z_2}$

② uniform diameter ; Area C/s : Same.
$A_1 = A_2 \Rightarrow Q = C$ ∴ $\boxed{V_1 = V_2}$.

③ Given Head difference ;
$E_1 = E_2 + h_{loss} \Rightarrow \boxed{h_{loss} = E_1 - E_2}$
$\Rightarrow h_{loss} \; 1^{st} = h_{loss} \; 2^{nd}$

GATE-07 CIVIL | Water flows through a pipe AB of 10 m length; uniform cross section area inclined at 30 with horizontal for a pressure of 12 kN/m² at the end B, the corresponding pressure P_A = (?) ; neglect losses

$P_A = (?)$
$P_B = 12$ kN/m²
$z_B = 5$ m
$z_A = 0$

$E_A = E_B$

$$z_A + \frac{P_A}{\rho g} + \frac{V_A^2}{2g} = z_B + \frac{P_B}{\rho g} + \frac{V_B^2}{2g}$$

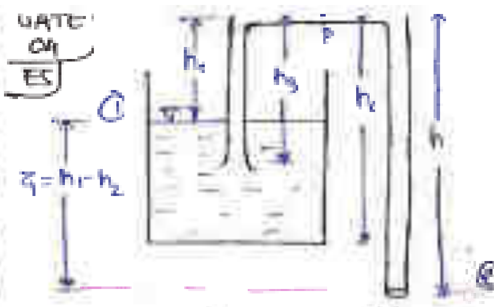
$Q = C \Rightarrow A_1 = A_2$
so ; $V_A = V_B$

$$\frac{P_A}{\rho g} = 5 + \left(\frac{P_B}{\rho g}\right) = 5 + \frac{12 \times 10^3}{1000 \times 9.81}$$

$= 1.22 \times 1000 \times 9.81$

$\boxed{P_A = 61.05 \text{ kPa}}$

GATE ON ES



A syphon draws a water from a large reservoir & release into atmosphere as shown. Find velocity at point 'P' (2)

- Ideal gas
- z_1 & z_2 are given to form of Bernoulli's eq^n
- at open surface atmospheric P_1
- of Large reservoir we consider velocity zero area large

at symphon $E_1 = E_2$

$$\left[z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g}\right]_① = \left[z_2 + \frac{P_2}{\rho g} + \frac{V_2^2}{2g}\right]_②$$

$$(h_1 - h_2) + \frac{P_{atm}}{\rho g} + 0 = 0 + \frac{P_{atm}}{\rho g} + \frac{V^2}{2g}$$

$$\boxed{V = \sqrt{2g(h_1 - h_2)}}$$

NOTE :

$\boxed{V_{approach} = 0}$

Area is large at time velocity too low $A_1 \uparrow\ V_1 \downarrow$ so $V = 0$

$A \uparrow V \downarrow = A \downarrow V \uparrow$

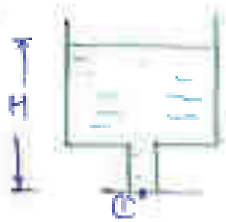
so, $\underbrace{z_1 + \frac{P_1}{\rho g}}_{100} + \underbrace{\frac{V_1^2}{2g}}_{0} = 100$ so $\Rightarrow 100 + 0 = 100$

NOTE :

When H is given reservoir is not too large

$\boxed{V = \sqrt{2gH}}$

NOTE :

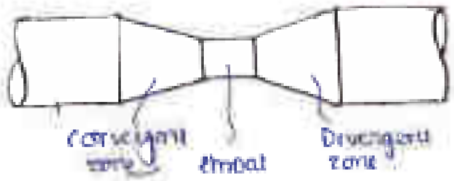


$\boxed{V = \sqrt{2gH}}$

→ **Application :**

(1) **Venturimeter** :- to measure flow rate / discharge

principle : By varying area of flow pressure difference is created & that is measured applying Bernoulli's equation. flowrate can be estimated.



NOTE : - pressure head in throat is minimum

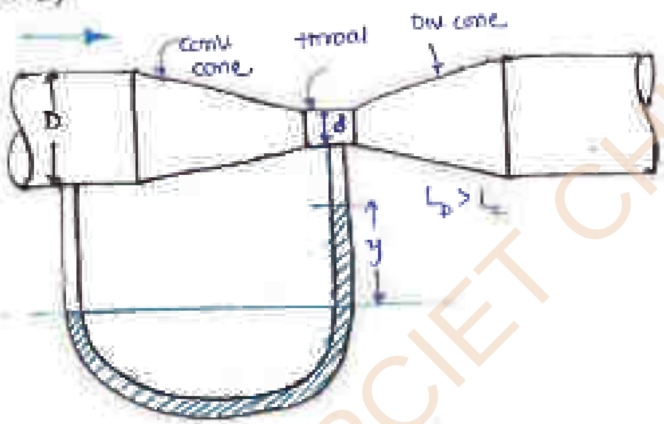
$Q = A\downarrow v\uparrow$

$z_1 + \frac{p\downarrow}{\rho g} + \frac{v^2\uparrow}{2g} = \text{const}$

(for venturimeter) → Coefficient discharge $\boxed{C_d = 0.98}$

→ $C_d = \frac{Q_{act}}{Q_{th}}$

(for orificemeter) → coefficient discharge $\boxed{C_d = 0.62}$



→ Convergent angle $= 20^\circ \pm 2^\circ$ ✓

→ Divergent angle $= 5^\circ$ to 7° ✓

→ Length of div > Length of conv.

→ dia of throat $\boxed{d = \frac{D}{2} \text{ to } \frac{D}{3}}$

NOTE (1) pressure at throat is minimum. (less than atmospheric) & it should fall below vapour pressure to avoid cavitation

(2) The divergence angle is limiting bet^n 5° to 7° to ensure smooth flow. [to avoid separation]

(c) The diameter of throat governed by vapour pressure of flowing fluid.
as y value increases error decreases

→ **Application of Bernoulli's eq^n**

→ Apply Bernoulli's eq^n @ (1) & (2)

$$z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$\underbrace{\left[z + \frac{P}{\rho g}\right]_1 - \left[z + \frac{P}{\rho g}\right]_2}_{\text{Venturi Head 'H'}} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \frac{V_2^2 - V_1^2}{2g}$$

$\{Q = AV$

$$A_1V_1 = A_2V_2 = Q \Rightarrow \left(\frac{Q}{A_2}\right)^2 - \left(\frac{Q}{A_1}\right)^2 = 2gH$$

$$Q^2\left[\frac{1}{A_2^2} - \frac{1}{A_1^2}\right] = 2gH$$

$$\boxed{Q_{th} = \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gH}}$$

but $C_d = \frac{Q_{act}}{Q_{th}} \Rightarrow Q_{act} = C_d\, Q_{th}$

$$\boxed{Q_{act} = \frac{C_d\, A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gH}}$$

→ Venturi Head

$$\boxed{H = \left[z_1 + \frac{P}{\rho g}\right]_1 - \left[z + \frac{P}{\rho g}\right]_2}$$

$$\boxed{H = y\left[\frac{S_m}{S_o} - 1\right]} \text{ m of flowing fluid}$$

for $[S_m > S_o]$

$P_A - P_B = \gamma_w\, y(S_m - S_o)$ N/m^2 (x)

$= y(S_m - S_o)$ m of water (x)

$= y\left[\frac{S_m}{S_o} - 1\right]$ m of flowing fluid (✓)

If $S_m < S_o$ ⇒ $H = y\left[1 - \frac{S_m}{S_o}\right]$

$$Q_{act} = \frac{C_d\, A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gH}$$

$$\boxed{Q_{act} = C_d\, K\sqrt{H}}$$

where ; K = venturi const

$= \frac{A_1A_2}{\sqrt{A_1^2 - A_2^2}}\sqrt{2g}$ (unit → $\frac{m^{2.5}}{sec.}$

If C_d is not given & asking Q_{act} ⇒ $\boxed{C_d = \sqrt{\frac{H - h_{L}}{H}}}$

NOTE: ① If there is an x % error in the venturi meter measurement; the corresponding error in measurement of flow rate x/2 %.

x % error ⟶ Head measurement

Q flow rate ⟶ x/2 % ⟶ $\left[\frac{dQ}{Q}\right]$

⟶ $Q \propto \sqrt{H} \Rightarrow Q = C\sqrt{H}$ — (1)

$dQ = C \cdot \frac{1}{2\sqrt{H}} \cdot dh$ — (2)

$\frac{dQ}{Q} = \frac{C \cdot \frac{1}{2\sqrt{H}} dh}{C\sqrt{H}} = \frac{1}{2}\left(\frac{dh}{H}\right)$

$$\boxed{\frac{dQ}{Q} = \frac{1}{2}\left(\frac{dh}{H}\right)}$$

Application:

- Orificemeter: To measure the flowrate / discharge

- To measure the flow rate / discharge
- Approximate value of $C_d = 0.63$ to 0.67
- Large dia. pipe

where: A_1 = Area of main pipe

A_0 = Area of orificemeter

A_2 = The min. area at veena contracta

Coefficient of contraction 'C_c'

$C_c = \frac{A_2}{A_0} \Rightarrow A_2 = A_0 \times C_c$

$$\boxed{Q_{act} = \frac{C_d A_1 A_2}{\sqrt{A_1^2 - A_2^2}} \sqrt{2gH}}$$

but $\Rightarrow K = \dfrac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2g}$ $\dfrac{m^{2.5}}{sec.}$ (unit)

$$\boxed{Q = C_d K \sqrt{H}}$$

& $H = y\left[\dfrac{S_m}{S_0} - 1\right]$ (Pressure Head difference)

$$= \left[z + \frac{p}{\rho g}\right]_1 - \left[z + \frac{p}{\rho g}\right]_2$$

GATE-14 | For a flow of water through a circular pipe of 30 cm dia. connected with venturimeter of 15 cm using manometric fluid of mercury with a $C_d = 0.98$ & manometric fluid deflection 10 cm. Estimate flowrate through pipe line is

→ $A_1 = \pi/4\,(30)^2 = 706.5\ cm^2$;

$A_2 = \pi/4\,(15)^2 = 176.625\ cm^2$

$$Q_{act} = \frac{C_d\, a_1 a_2}{\sqrt{a_1^2 - a_2^2}} \sqrt{2gH}$$

so, $H = y\left[\dfrac{S_m}{S_0} - 1\right] = 10\left[\dfrac{13.6}{1} - 1\right] = 126 = 1.26$

$$Q_{act} = \frac{(0.98)(706.5)(176.625)}{\sqrt{706.5^2 - 176.62^2}} \sqrt{2 \times 9.81 \times 126}$$

$$= \frac{(0.98)(0.0706)(0.0176)}{\sqrt{0.0706^2 - 0.0176^2}} \sqrt{2 \times 9.81 \times 1.26}$$

$$\boxed{Q_{act} = 0.088\ m^3/s}$$

GATE-14 | For a flow of water through a circular pipe connected with venturimeter with a constant of $0.3\ m^{2.5}/sec$ at $C_d = 0.96$ using mercury has manometric fluid & estimate flow rate for pressure Head difference of 50 cm of Hg (H)

→ $H = 50$ cm of Hg $= \left[z + \dfrac{p}{\rho g}\right]_1 - \left[z + \dfrac{p}{\rho g}\right]_2 = 0.5$ m of Hg

$C_d = 0.96$

$K = 0.3\ m^{2.5}/sec$

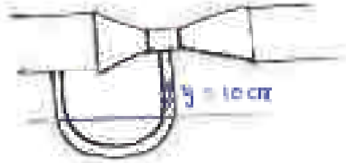
$Q = C_d K \sqrt{H}$ — m of flowing fluid

Head difference
$= 0.5$ m of Hg
$= 0.5 \times S_{Hg}$ (m of flowing f.)

But $H = 0.5$ m of Hg
$= 0.5 \times 13.6$
$= 6.8$ m of flowing fluid

$Q = 0.96 \times 0.3 \sqrt{6.8}$ ⇒ $\boxed{Q = 0.75\ m^3/sec}$

GATE-14 (civil)



If tilt to 45° then for const Q = C, deflection value y = ?

✓ y = 10 cm

$$Q = C_d\, K \sqrt{H} = \text{const}$$

(C_d const, K const, $\sqrt{H}$ const)

$$H = \left[z + \frac{P}{\rho g}\right]_1 - \left[z + \frac{P}{\rho g}\right]_2 = \text{const}$$

$$= y\left[\frac{S_m}{S_0} - 1\right] \quad \text{const}$$

($y \to$ const)

ES: Flow of circular pipe of 30 cm diameter inclined at 30° connected with venturimeter of 10 cm dia. using mercury as manometric fluid & with deflection of 10 cm. Estimate the approximate flow rate by considering the loss across venturimeter 20% of K.E. at inlet.

→ $a_2 = 0.0706$

$a_1 = 0.1962$

$Q_{appro} = ?$

$y = 10\text{ cm} = 0.1$ m; $H = y\left[\frac{S_m}{S_0} - 1\right]$

$= 1.26$ m of Δg

Gate: w/o find C_d?

$$C_d = \sqrt{\frac{H - h_{loss}}{H}} = \sqrt{\frac{1.26 - (0.2)V_1^2/2g}{1.26}}$$

(we can't find w/o V given, so take $C_d = 0.96$ (GATE))

$$h_{loss} = 20\%(K.E.)_{inlet}$$

$$= 0.2\left(\frac{V^2}{2g}\right)_1 \quad \text{+ neg}^n\ V_1^* \text{ value}$$

$$Q_{app} = \frac{(0.96)(0.1962)(0.0706)\sqrt{2 \times 9.81 \times 1.26}}{\sqrt{0.1962^2 - 0.0706^2}}$$

$$\boxed{Q_{app} = 0.3665\ m^3/s}$$

ES: find C_d: (Full solution)

$$E_1 = E_2 + h_{loss}$$

$$\left[z + \frac{P}{\rho g} + \frac{V^2}{2g}\right]_1 = \left[z + \frac{P}{\rho g} + \frac{V^2}{2g}\right]_2 + 0.2\frac{V_1^2}{2g}$$

$$\left[z + \frac{P}{\rho g}\right]_1 - \left[z + \frac{P}{\rho g}\right]_2 = \frac{V_2^2}{2g} + \frac{0.2V_1^2}{2g} - \frac{V_1^2}{2g} = \frac{V_2^2 - 0.8V_1^2}{2g}$$

$$\left[z+\frac{P}{\rho g}\right]_1-\left[z+\frac{P}{\rho g}\right]_2=H=y\left[\frac{S_m}{S_0}-1\right]=\frac{V_2^2-0.8V_1^2}{2g}$$

$$\therefore \frac{V_2^2-0.8V_1^2}{2g}=y\left[\frac{S_m}{S_0}-1\right]$$

$$\therefore \left(\frac{Q}{A_2}\right)^2-0.8\left(\frac{Q}{A_1}\right)^2=2gy\left[\frac{S_m}{S_0}-1\right]$$

$$Q^2=\frac{2gy\left[\frac{S_m}{S_0}-1\right]}{\frac{1}{A_2^2}-\frac{0.8}{A_1^2}}$$

$$Q=\sqrt{\frac{2gy\left[{S_m}/{S_0}-1\right]}{1/A_2^2-0.8/A_1^2}}=\sqrt{\frac{2\times9.81\times1.26}{(1/0.0706)^2-(0.8/0.1963)^2}}$$

$\boxed{Q_{th}=0.376\ m^3/s}$ by considering C_d zero/unity.

GATE-ME / AIS) For a measurement of flow of water through a circular pipe using venturimeter with $C_d=0.98$ was replaced by orificemeter of $C_d=0.63$ for the same flow parameter the ratio of pressure drop across the the venturi to orificemeter.

→ $Q_v=Q_o$ $\boxed{H = \text{Pressure Head difference} = \Delta P}$

$C_{d_v}K\sqrt{H_v}=C_{d_o}K\sqrt{H_o}$

$$\boxed{\frac{H_v}{H_o}=\frac{0.63^2}{0.98^2}}$$

$$\frac{\Delta P_{venturi}}{\Delta P_{orifice}}=\frac{0.63^2}{0.98^2}$$

→ If y in venturimeter is $= 60$ cm then; y in orificemeter $=$?

$y=H\left[\frac{S_m}{S_0}-1\right]$ → $y\propto H$

$$\frac{y_{venturi}}{y_{orifice}}=\frac{H_{venturi}}{H_{orifice}}=\frac{0.63^2}{0.98^2}=\frac{C_d^2}{C_d^2}$$ so $\boxed{y_{orifice}=24.1\ cm}$

NOTE

$$\boxed{y\propto\frac{1}{C_d^2}\propto H\propto P}$$

$$Q=C_dK\sqrt{y\left[\frac{S_m}{S_0}-1\right]}=C_dK\sqrt{H}$$

$$Q\propto\frac{1}{\sqrt{y}}\propto\sqrt{y}\propto\frac{1}{C_d}\propto\sqrt{H}$$

(e) Pitot tube

→ To measure velocity

It is point where partical impact → $V=0$ → [stagnation pt.] (flow not moving) - entire velocity zero so the energy do not destroy so it converted into pressure Head $[\frac{P}{\rho g}]$ at pt.

$$\left[z + \frac{P}{\rho g}\right] + \frac{V^2}{2g} = 100$$

(b) $60 + 40 = 100$

h_{static}

(c) But at stagnation $V=0$ ... converted

$100 + 0 = 100$

h_{stg}.

(d) But convert entry in to K.E → h_{dy}

$60 + 40 = 100$

h_{dyn}

$$\boxed{h_{stg} = h_{static} + h_{dynamic}}$$

(a) velocity
(b) static pressure head
(c) Dynamic pressure Head
(d) stagnation pressure Head

→ Pitot tube measure stagnation pressure

→ in velocity $V = \sqrt{2gh}$; h is dynamic Head

$$\boxed{V = \sqrt{2g\, h_{dynamic}}}$$

Apply Bernoulli's eqn ① & ②

$$\left(z + \frac{P}{\rho g} + \frac{V^2}{2g}\right)_1 = \left[z + \frac{P}{\rho g} + \frac{V^2}{2g}\right]_2$$

$$\underbrace{\left[z + \frac{P}{\rho g}\right]_1}_{h_{stat}} + \frac{V^2}{2g} = \underbrace{\left[z + \frac{P}{\rho g}\right]_2}_{h_{stg}} + \cancel{\frac{V_2^2}{2g}}^{\,0}$$

$$\frac{V^2}{2g} = h_{stg} - h_{stg} = h_{dynamic}$$

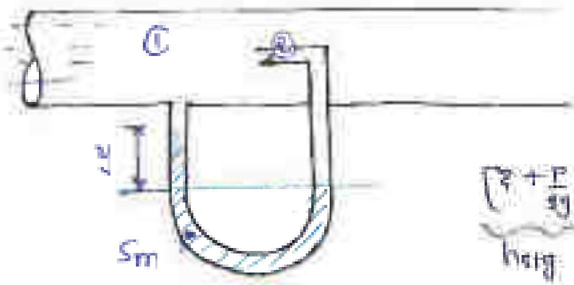
$$\boxed{V_{th} = \sqrt{2g\, h_{dynamic}}}$$

But $C_v = \frac{V_{act}}{V_{th}}$ ⇒ $\boxed{V_{act} = C_v \sqrt{2g\, h_{dynamic}}}$

$$\boxed{C_d = C_c \times C_v}$$

⟹ Measurement of gas velocity : [Pitot - static tube]

- above inverted mano not possible becoz not fluid having density low than air ; Air has low ρ ; against gravity not possible.
- Gas is incompressible fluid ; [M < 0.4]
- If gas velocity measure by pitot - static tube



$$\underbrace{\left[z + \frac{P}{\rho g}\right]_2}_{h_{stg}} - \underbrace{\left[z + \frac{P}{\rho g}\right]_1}_{h_{stc}} = h_{dyna.}$$

$$= y\left[\frac{S_m}{S_0} - 1\right]$$

$$\boxed{V = C_v\sqrt{2gy\left[\frac{S_m}{S_0} - 1\right]}}$$

S_0 → S of flowing fluid (main pipe) ; $S = \frac{\rho \text{ of any}}{\rho_{H_2O}} = \frac{\rho_{air}}{\rho_{H_2O}}$

$$\boxed{V = C_v\sqrt{2gy\left[\frac{S_m}{S_0} - 1\right]}}$$

- pitot tube measure V_{mean} (mean velocity)

Ex For measurement of velocity of water through a pipe line pitostatic tube with coefficient of 0.98 ; the stagnation pressure head was obtained as 3 m ; static pressure head 0.5 m the approximate flow of velocity will be

→ $h_{dynamic} = h_{static} - h_{stg.}$

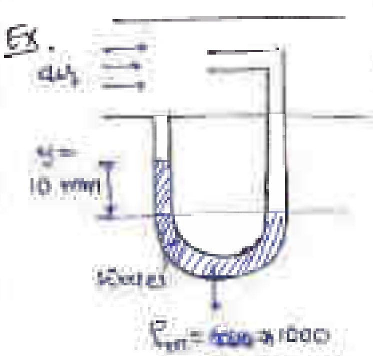
$= 3.0 - 0.5$

$h_{dyn.} = 2.5$

$V = C_v\sqrt{2g\,h_{dyn.}} = 0.98\sqrt{2 \times 9.81 \times 2.5}$

$$\boxed{V = 6.8 \text{ m/s}}$$

Ex.



$\rho_{air} = 1.2 \text{ kg/m}^3$; $V_{gas} = ?$; $\rho_0 = 1.2 \text{ kg/m}^3$

$\rho_m = 1000 \text{ kg/m}^3$ (water)

$V = C_v\sqrt{2gy\left[\frac{S_m}{S_0} - 1\right]}$

$= C_v\sqrt{2gy\left[\frac{\rho_m}{\rho_0} - 1\right]}$ — ρ of air (flowing in main pipe)

$= (1)\sqrt{2 \times 9.81 \times 0.01\left[\frac{1000}{1.2} - 1\right]}$

$$\boxed{V = 12.77 \text{ m/s}}$$

Ex For measurement of flow of water using pressure tube flow air $\rho_{air} = 1.2\ kg/m^3$ (ρ_0) with manometric fluid of density 800 kg/m^3 the d/H^n in stagnation & static pressure is estimated as 350 Pa. $V_{flow}(air) = ?$

→ $P_{stg} - P_{static} = 350\ N/m^2$

$$\underbrace{z_1 + \frac{P}{\rho g}}_{P_{sta}} + \frac{V_1^2}{2g} = \underbrace{z_2 + \frac{P_2}{\rho g}}_{P_{stg}} + \frac{V_2^2}{2g}$$

$$P_{dyna} = \frac{V_1^2}{2g} = P_{stg} - P_{stg} = 350\ N/m^2$$

h_{dyna} = 350 × ...

$$\boxed{P_{dynamic} = 350\ N/m^2}$$

$$h_{dyna} = \frac{P_{dyn}}{\rho g} = \frac{P_{dyn}}{\rho_{air} g} = \frac{350}{1.2 \times g}$$

where ρ → flowing fluid (ρ air)

$$V = C_v \sqrt{2g\, h_{dyn}} = 1.0 \sqrt{2g \times \frac{350}{1.2 \times g}}$$

$$\boxed{V_a = 25.5\ m/s}$$

GATE-12

a) $\sqrt{2g(h_1 + h_2 + h_3)}$

b) $\sqrt{2g\left[\frac{\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3}{\rho_1 + \rho_2 + \rho_3}\right]}$

c) $\sqrt{2g h_3\left[1 + \frac{\rho_1 h_1}{\rho_3 h_3} + \frac{\rho_2 h_2}{\rho_3 h_3}\right]}$ ✓

d) ×

Apply Bernoulli's eq^n ① & ②

$$\left[z + \frac{P}{\rho g} + \frac{V^2}{2g}\right]_1 = \left[z + \frac{P}{\rho g} + \frac{V^2}{2g}\right]_2$$

Here it is Large Reservoir at 1 so $V = 0$

flowing fluid ρ_3

$$\left[\frac{\rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3}{\rho_3 g}\right] + 0 = 0 + \frac{V^2}{2g}$$

$$V^2 = \frac{2g[\rho_1 h_1 + \rho_2 h_2 + \rho_3 h_3]}{\rho_3} \times \frac{h_3}{h_3}$$

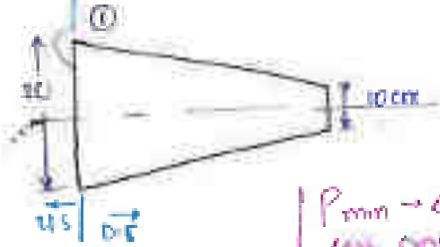
$$\boxed{V = \sqrt{2g h_3\left[1 + \frac{\rho_1 h_1}{\rho_3 h_3} + \frac{\rho_2 h_2}{\rho_3 h_3}\right]}}$$

where diameter reduces from 20 cm to 10 cm in Horizontal pipe line. The pressure in 20 cm pipe just upstream of reducer is 150 kPa & the vapour pressure is 10 kPa. & 7.5 kN/m³ : estimate the max possible flow rate that can pass through reducer without causing cavitation.

$a_1 = \frac{\pi}{4}(0.2)^2 = 0.0314 \quad \gamma = 5\ \frac{kN}{m^3}$

$a_2 = \frac{\pi}{4}(0.1)^2 = 0.00785$

$P_1 = 15\ kPa = 15\times10^3\ N/m^2$



$P_{min} \to$ at least cross section

$$\cancel{Z_1} + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \cancel{Z_2} + \frac{P}{\rho g} + \frac{V_2^2}{2g}$$

so, with cavitation $P_{min} > P_{vapour}$

so, $P_2 = P_{min} = 50\times10^3\ N/m^2 > P_v$

$$\frac{150\times10^3}{5\times10^3} + \frac{V_1^2}{2g} = \frac{50\times10^3}{5\times10^3} + \frac{V_2^2}{2g}$$

$$\frac{V_2^2 - V_1^2}{2g} = \frac{100}{5} = 20$$

$$V_2^2 - V_1^2 = 20\times2\times9.81$$

but $Q = A_1V_1 = A_2V_2 \Rightarrow 0.0314\,V_1 = 0.00785\,V_2$

$V_2 = 4V_1$

$$(4V_1)^2 - V_1^2 = 2\times20\times9.81$$

$$V_1 = \sqrt{\frac{2\times20\times9.81}{15}}$$

$$\boxed{V_1 = 5.11\ m/s}$$

$Q_1 = A_1V_1 \Rightarrow \boxed{Q = 0.16\ m^3/s}$

$\Rightarrow$ convert problem in venturimeter

$$Q_{th} = \frac{a_1 a_2}{\sqrt{a_1^2 - a_2^2}}\sqrt{2gH}$$

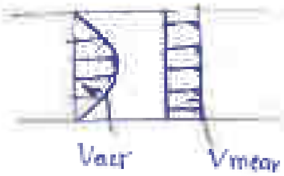
but $H = \left[z_1 + \frac{P}{\rho g}\right]_1 - \left[z + \frac{P}{\rho g}\right]_2$

$= \frac{150}{5} - \frac{50}{5} = 20$

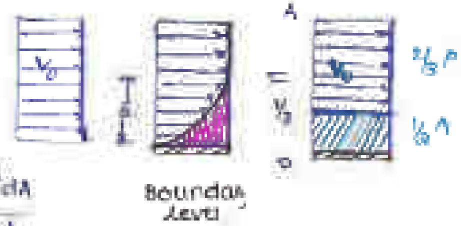
$\boxed{Q_{th} = 0.16\ m^3/s}$

$$\alpha = \frac{(KE)_{act}}{(KE)_{mean}} \quad \overline{A V^3_{mean}}$$

Ex. For a fluid flowing through a pipeline, the velocity profile was approximated as zero over 1/3rd of area & that remain constant over the rest of area. Then (K.E) correction factor value is



zero → 1/3rd area → C
const → 2/3rd → V



$$\alpha = \frac{(KE)_{act}}{(KE)_{mean}} = \frac{\int V_{act}^3 dA}{V_{m}^3 A} = \frac{\int V^3 dA}{V_m^3 A}$$

$$\int V^3 dA = V_{act} = \int_{1/3 A}^{A} (V)^3 dA + \int_{0}^{A/3} 0^3 dA = V^3\left[A - \frac{A}{3}\right]$$

$$\int V^3 dA = \frac{2AV^3}{3}$$

$$V_{mean} = \frac{Q}{A} = \frac{\int V\, dA}{A}$$

$$\int V dA = \int_{0}^{A/3} 0\, dA + \int_{A/3}^{A} V\, dA = V\left[A - \frac{A}{3}\right] = 2AV/3$$

$$V_{mean} = \frac{2AV}{3A} = \frac{2V}{3}$$

so: $\alpha = \dfrac{2/3\, A V^3}{A\left(\frac{2}{3}V\right)^3} = \frac{2}{3}\left(\frac{3}{2}\right)^3 = \boxed{9/4}$

→ Instrument → uses

Instrument		uses	
Venturimeter, Orificemeter	}	flow rate	← $[\Delta P/\rho g]$
Pitot tube	→	velocity	← h_{dyn} ← $[h_{sq}]$
Rota meter	→	velocity flow rate	
Hot-wire Anemometer	→	turbulent vel. fluctuation	
Hydrometer	→	specific gravity	← [Buoyancy principle]
Hygrometer	→	moisture indicator [Humidity]	
Current meter	→	velocity	

$$Re = \frac{F_i}{F_v} = \frac{\text{inertia force}}{\text{viscous force}} = \frac{\rho V L}{\mu}$$

If nothing mention, take circular pipe

	circular pipe	Betⁿ parallel plates	plate open channel	over a sph. through soil/p
Laminar	$Re < 2000$	$Re < 1000$	$Re < 500$	$Re < 1$
Transitional	$2000 < R_e < 4000$	$1000 < R_e < 2000$	$500 < R_e < 1000$	$1 < R_e < 2$
Turbulent	$Re > 4000$	$Re > 2000$	$Re > 1000$	$Re > 2$

critical zone; critical

$E_1 = E_2 + h_{loss}$

Major [60 - 90%] → friction

Minor [10 - 20%] → Pipe c/s variation, External fitting.

Minor loss :

① h_L due to sudden contraction

velocity at contraction

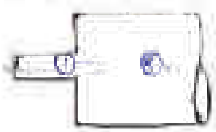
$$\boxed{h_L = \left[\frac{1}{C_c} - 1\right]^2 \frac{V_2^2}{2g}}$$

where ; C_c = coefficient of contraction

If C_c is not given ;

$$\boxed{h_L = 0.5\frac{V_2^2}{2g}}$$

② h_L due to sudden Expansion :



$$\boxed{h_L = \frac{(V_1 - V_2)^2}{2g}}$$

③ h_L at entry

velocity at entry

$$\boxed{h_L = \frac{0.5 V^2}{2g}}$$

$$h_L = \frac{V^2}{2g}$$ velocity at exit

ⓒ

$$\boxed{h_L = K\left[\frac{V^2}{2g}\right]}$$ Where ; K = loss coefficient

★★★★★ Frictional loss :-

Darcy Weisbach's Equation

$$\boxed{h = \frac{4fLV^2}{2gd}}$$

Where, V = Vmean

f = frictional coefficient

= 0.005 to 0.01

4f = F

= frictional factor

= 0.02 to 0.04

modified Darcy eq^n :

$$\boxed{h = \frac{FLV^2}{2gd}}$$

- By seeing range of f decide to take 'f' or 'F'

- Friction value may consider can from moody chart

Re ≤ 2000 $F = \frac{64}{Re}$ → Laminar

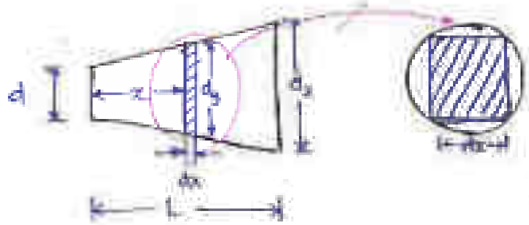
R > 4000 $F = \frac{0.316}{(Re)^{1/4}}$ → turbulent

For flow : $Q = AV \Rightarrow V = \frac{Q}{\frac{\pi}{4}d^2}$

$$h = \frac{FLV^2}{2gd} = \frac{FL}{2gd}\left(\frac{Q}{\frac{\pi}{4}d^2}\right)^2$$

★★ $$\boxed{h_f = \frac{FLQ^2}{12.1\,d^5}}$$

Special case - (i)



where $F = F$
$L = dx$
$Q = Q$
$d = d_y$

so: $h_f = \int dh_f$
$= \int \frac{FLQ^2}{12.1 d^5}$
$= \int \frac{F(dx)Q^2}{12.1 d_y^5}$

but $d_y = d_1 + \left(\frac{d_2 - d_1}{L}\right)x$

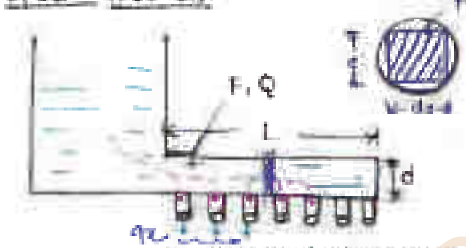
so, $h_f = \frac{F}{12.1} Q^2 \int_{x=0}^{L} \frac{dx}{\left[d_1 + \left(\frac{d_2-d_1}{L}\right)x\right]^5}$

$= \frac{F}{12.1} Q^2 \int_0^L \frac{dx}{[A + Bx]^5}$

$= \frac{F}{12.1} Q^2 \left[\frac{(A+Bx)^{-4}}{-4}\right]_0^L = \frac{-F}{(12.1)(4)} Q^2 \left\{\left[d_1 + \left(\frac{d_2-d_1}{L}\right)L\right]^{-4} - [d_1 + 0]^{-4}\right\}$

$h_f = \frac{FQ^2}{48.4}\left[\frac{1}{d_1^4} - \frac{1}{d_2^4}\right]\left[\frac{L}{d_2 - d_1}\right]$

Special case (ii)



q = lateral discharge per unit length

$$\boxed{h_f = \frac{1}{3}\left(\frac{FLQ^2}{12.1 d^5}\right)}$$

$$\boxed{Q = q \cdot L}$$

- Here discharge is varying becoz of 100 times passes 90, 80, 70 ... 90

- Here length is varying across flow through c/s, so take element of uniform length

so, $h_f = \int dh_f = \int \frac{F(dx)(Q')^2}{12.1 d^5}$

$Q' = Q - qx = q[L - x]$

$$\boxed{h_f = \frac{1}{3}\left(\frac{FLQ^2}{12.1 d^5}\right)}$$

$$\boxed{V = C\sqrt{mi} = C\sqrt{RS}}$$

where V = V_{mean}

C = Chezy's constant $\rightarrow$ $\boxed{C = \sqrt{\frac{8g}{F}}}$

i, S = Hydraulic slope / gradient $= \left[\frac{h_f}{L}\right]$

m (or) R = ~~mean~~ Hydraulic mean depth / radius

$$= \frac{\text{Area of flow}}{\text{wetted perimeter}} = \frac{A}{P}$$

→ for circular pipe ; Running full

$$m = \frac{A}{P} = \frac{\frac{\pi}{4}d^2}{\pi(d)} = \frac{d}{4}$$

$$m = d/4$$

→ for circular pipe ; Running half full

$$m = A/P = \frac{(\frac{\pi}{4}d^2)\frac{1}{2}}{(\pi d/2)} = d/4$$

→ $v = C\sqrt{mi}$

$$v^2 = c^2 (m)(i)$$

$$= \frac{8g}{F}v^2 \cdot \frac{d}{4} \cdot \frac{h_f}{L}$$

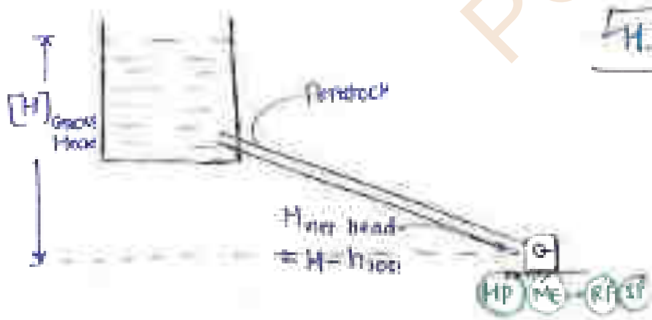
$$= \frac{8g}{F}c^2$$

$$\boxed{C = \sqrt{\frac{8g}{F}}}$$

Power Transmission :-

Hydraulic Power / [water power]
{Net Available}

$$\boxed{H.P = \gamma Q[H - h_f]}$$

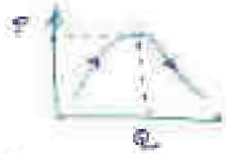


Loss in power
frictional power
pumping power $\Big\} = \gamma Q h_f$ where: $h_f = \frac{FLQ^2}{12.1d^5}$

$$\eta = \frac{\gamma Q[H-h_f]}{\gamma Q H} \Rightarrow \boxed{\eta = \frac{H-h_f}{H}}$$

PSU → condition for P_{max} : $h_f = H/3$

$$\boxed{\eta_{\substack{max\\power}} = 66.67\%}$$



[*] condition for maximum power

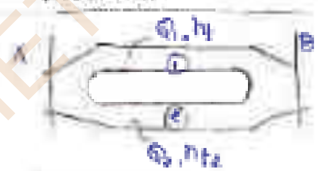
$$P = \gamma Q[H - h_f]$$
$$= \gamma Q\left[H - \frac{FLQ^2}{12.1d^5}\right]$$

→ $Q\uparrow \rightarrow P\uparrow$
$Q\uparrow \rightarrow [H - h_f\downarrow] \rightarrow P\downarrow$
so, Q variable

$$\frac{dP}{dQ} = 0 \Rightarrow \frac{d}{dQ}\left[\gamma\left[HQ - \frac{FLQ^3}{12.1d^5}\right]\right] = 0$$
$$\gamma\left[H - 3\left(\frac{FLQ^2}{12.1d^5}\right)\right] = 0$$
$$H - 3h_f = 0$$
$$\boxed{h_f = \frac{H}{3}}$$ available Gross Head

→ $\eta_{@\,\substack{max\\power}} = 66.67\%$

★ Pipe connection :

Series	Parallel
	
$Q_{AB} = Q_1 = Q_2$	$Q_{AB} = Q_1 + Q_2$
$h_{f_{AB}} = h_{f_1} + h_{f_2}$	$h_{f_{AB}} = h_{f_1} = h_{f_2}$

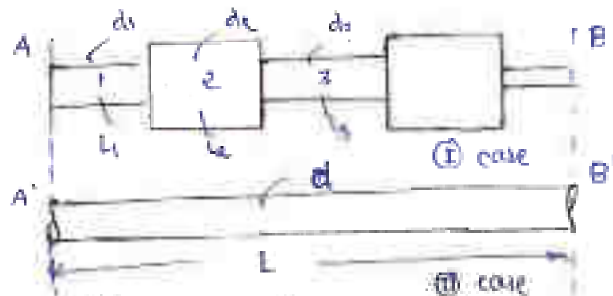
→ $E_A = E_B + h_{f_1} \Rightarrow E_A - E_B = h_{f_1}$
→ $E_A = E_B + h_{f_2} \Rightarrow E_A - E_B = h_{f_2}$

$$h_{f_{1st}} = h_{f_{2nd}}$$

$$\frac{FLQ^2}{12.1 D^5} = \frac{FL(Q/n)^2}{12.1 d^5} \Rightarrow \boxed{d = \frac{D}{n^{2/5}}}$$

- If flowing fluid in same diameter 'd' fluid from 2-pipes then flow rate = $Q/2$

→ **Dupit's Equation.**



- municipal water supply water system with diffⁿ size

∴ h_{loss} ① case $= h_{loss}$ ② case

$$[h_{f_1} + h_{f_2} + h_{f_3} + \dots + h_{f_n}]_{1st} = [h_f]_{2nd}$$

$$\frac{FL_1Q^2}{12.1 d_1^5} + \frac{FL_2Q^2}{12.1 d_2^5} + \dots = \frac{FLQ^2}{12.1 d^5}$$

$$\boxed{\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \dots = \frac{L}{d^5}}$$

GATE 2009) Water at 25°C flowing through 1 km long G.I. pipe of 200 mm diameter at a flow rate of 0.07 m³/s, $f = 0.02$; Pumping power reqⁿ in kW = (?)

$Q = 0.07$
$L = 1000$ mm
$d = 0.2$
$\gamma_w = 9810$ N/m³
$F = 0.02$

If F is 0.02 to 0.04 range then it is F is given

Pumping Power $= \gamma Q [H - h_f]$ $\quad \left\{ h_f = \frac{FLQ^2}{12.1 d^5} \right.$

$= \gamma Q h_f$

$= (9810)(0.07)\left(\frac{FL\,Q^2}{12.1 d^5}\right)$

$= (9810)(0.07)\left(\frac{0.02 \times 1000 \times 0.07^2}{12.1 \times (0.2)^5}\right)$

$= 17.4$ kW

civil reservoir with head difference 8 m. F = 0.04 accounting for frictional entering & exit loss, the velocity of flow through pipe (?)

So, $E_1 = E_2 + h_{loss}$

$$\left[z + \frac{P}{\rho g} + \frac{V^2}{2g}\right]_1 = \left[z + \frac{P}{\rho g} + \frac{V^2}{2g}\right]_2 + h_f + h_{f\,entry} + h$$

@ Patm pressure
→ velocity also same

$$z_1 = z_2 + h_f + h_{entry} + h_{exit}$$

$$z_1 - z_2 = \frac{FLV^2}{2gd} + 0.5\frac{V^2}{2g} + \frac{V^2}{2g} = 8$$

$h_{entry} = 0.5\frac{V^2}{2g}$

$h_{exit} = \frac{V^2}{2g}$

$$\frac{V^2}{2g}\left[\frac{FL}{d} + 0.5 + 1\right] = 8$$

$$V = \sqrt{\frac{8 \times 2 \times 9.81 \times 0.2}{(0.04)(2000) + (1.5)(0.2)}}$$

$$\boxed{V = 0.625 \text{ m/s}}$$

GATE-19) Mech. Water coming out from the top of the tap opening th stream with dia of 20 mm & velocity of 2 m/sec moving downwards with $acc^n = g$: neglecting surface tension & curvature effect consider const P atm throughout stream the dia of stream @ 0.5 m below the tap approximate is

$d_1 = 20$ mm & $V_1 = 2$ m/sec

$Q = \pi/4\, d_1^2 \times V_1 = 6.28 \times 10^{-4}$ m³/s

→ $Q = A_1V_1 = A_2V_2$

→ Bernoulli's eq^n

$$z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g} = z_2 + \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

$$z_1 - z_2 + \frac{V_1^2}{2g} = \frac{V_2^2}{2g}$$

$$0.5 + \frac{(2)^2}{2 \times 9.81} = \frac{V_2^2}{2g}$$

$$\boxed{V_2 = 3.71 \text{ m/s}}$$

→ At tap open: at bottom dia is decreses

b/oz.

↓ $z + \left(\frac{P}{\rho g}\right) + \frac{V^2}{2g} = C$

Patm const

$Q = A_1V_1 = A_2V_2$

$= A\downarrow V\uparrow$

→ $Q = (20)^2(2) = (d)^2(3.71)$

$$\boxed{d = 14.7 \text{ mm}}$$

whose length are 1200, 750 & 600 m & corresponding dia. of 750, 600 & 450 mm ; transform the system into equivalent 450 dia. pipe.

$$\frac{L_1}{d_1^5} + \frac{L_2}{d_2^5} + \frac{L_3}{d_3^5} = \frac{L_e}{d^5}$$

~~2133.53 + 2083.38 + 2162.96~~ $= \frac{L_e}{(0.45)^5}$

$$\frac{1200}{(750)^5} + \frac{750}{(600)^5} + \frac{600}{(450)^5} = \frac{L_e}{(450)^5}$$

$$\boxed{L_e = 671.3 \text{ m}}$$

ISRO -10) Pipe line connecting bet^n two reservoirs of its dia. reduced by 20% due to deposition of chemicals for a given head difference with unchanged friction factor. This would cause a reduction in flow rate of (?)

~~$Q_1 = A_1V_1 = A_2V_2 = Q_2$~~

$Q_1 \propto d_1^2 \propto d^2$ $\quad Q_2 \propto (0.8)^2 d^2$

$h_{f_1} = h_{f_2}$ (Head diff^n unchanged)

$$\frac{FLQ_1^2}{12.5 d_1^5} = \frac{FLQ_2^2}{12.5 d_2^5}$$

$$\frac{Q_1}{Q_2} = \left(\frac{d_1}{d_2}\right)^{5/2} = \left[\frac{d}{0.8d}\right]^{5/2} = 1.746$$

$\rightarrow$ % $\quad \left[\frac{Q_1 - Q_2}{Q_1}\right] \times 100 = \frac{1.746Q_2 - Q_2}{1.746 Q_2} = \underline{42.77\%}$

GATE -2014) Flow of water through circular pipe; of 10 cm diameter with velocity of 0.1 m/s ; $\nu = 10^{-5}$ m²/s & then the friction factor will be (?)

$$Re = \frac{\rho V d}{\mu} = \frac{0.1 \times 0.1}{10^{-5}} = \cancel{1.0\times10^{-3}}\ 1000$$

$F = \cancel{\frac{64}{R_e}}$ $\quad f = \frac{64}{R_e} = 0.064$ (Laminar)

$$\boxed{F = 0.064}$$

$F = 0.02$ to 0.04

$f = 0.005$ to 0.01

(correction)

but $\mu_w = 1.02 \times 10^{-3}$ NS/m²

$\nu = \mu/\rho = \frac{1.02\times10^{-3}}{10^3} = 10^{-6}$

$\Rightarrow Re = \frac{0.1\times10^{-1}}{10^{-6}} = 10100 > 4000$ (turbulent)

$F = \frac{0.316}{Re^{1/4}} = \frac{0.316}{(10^4)^{1/4}} \Rightarrow \boxed{F = 0.0316}$

- For flow of water through pipeline whenever valve is closed becoz of the disturbance of momentum of flowing fluid a pressure wave will generate & travells in opposite direction with accoustic speed. (sound speed) (c) by hearing the valve known as hammering effect.
- To avoid hammering effect surge tank will be used at the penstock.

Hammering ~~effect~~ velocity

$\boxed{c = \sqrt{K/\rho}}$ → Liquid

$\boxed{c = \sqrt{\gamma R T}}$ → Gas

where ; $\gamma = \frac{C_p}{C_v}$; c = speed

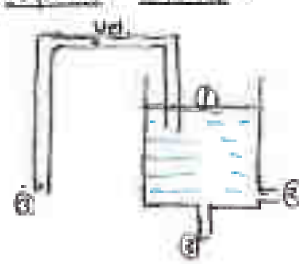
K = bulk modulus of liq^n

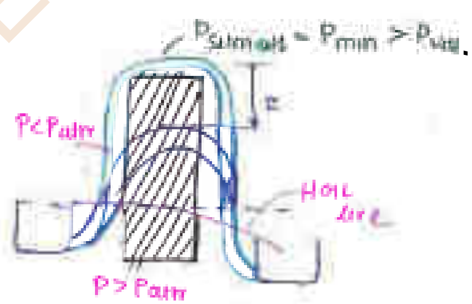
ρ = density of fluid.

→ Sudden closure : $t < \frac{2L}{c}$

→ Gradual closure : $t > \frac{2L}{c}$

(#) Syphon Effect :





$P_{summit} = P_{min} > P_{vapour}$

$\sigma > \sigma_c$

$\left(\frac{NPSH}{H}\right) > \sigma_c$

Laminar flow

$Re_{local} < Re_{critical}$

$\therefore NPSH > \sigma H$

Total Energy Line (T.E.L):

- It is line representing total available energy
- T.E.L will be horizontal for an ideal flow & and always slopes downward for real fluid flow.

Hydraulic Gradient Line (H.G.L)

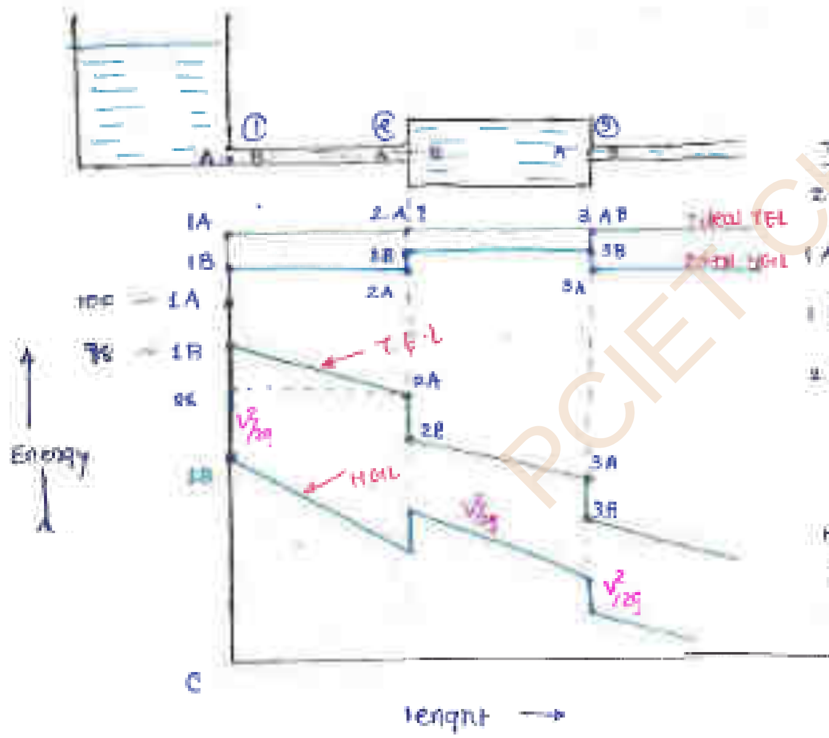
& also **piezometric line**

- H.G.L is line representing net available piezometric head $\left[z + \frac{P}{\rho g}\right]$.

$$\boxed{T.E.L - H.G.L = \frac{V^2}{2g}}$$

NOTE: ① H.G.L may rise or fall in the dir^n of flow;

② H.G.L represent atmospheric pressure condition for all the points above H.G.L pressure will be below atmospheric; for all the points below H.G.L the pressure will be above atmospheric. [For syphon flow]



TEL

$z + P/\rho g + V^2/2g = 100$

1 A — 100 + 0 = 100

h_{L} entry = 2

1 B — 60 + 15 = 75

$h_L = 10$

2 - B 50 + 15 = 65

$Q = A V$

HGL

$Q = AV$

$\left[z + \frac{P}{\rho g}\right] + \frac{V^2}{2g} = C$

$H \leftarrow \boxed{P = \gamma Q H}$

$E_1 - E_T = E_2 + h_{loss}$

- becoz turbine convert hydraulic energy into electrical energy so H.E is decrease

$E_1 + E_P = E_2 + h_{loss}$

- Pump convert elect. in Hydro. E that will added

GATE | Turbine working under head of 50 m the discharge in the feeding penstock 3 m^3/s considering a head loss across the runner of 5 m. the residual head in downstream of turbine, power = 1000 kW

→ power $= \gamma q H = \gamma Q H$

$1000 = 9810 \times 3 \times H$

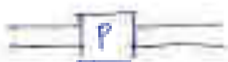
$\boxed{H = 33.97 \text{ m}} \leftarrow$ of E_T

→ $E_1 - E_T = E_2 + h_{loss}$

$50 - 35.97 = E_2 + 5$

$\boxed{E_2 = 11.02 \text{ m}}$ Residual head :

Ex) Pump; centrifugal pump 2 points A & B on suction & delivery pipe of the same size at same elevation with head loss of 3 m considering head develop by pump as 10 m for pressure of 120 kPa at pt B, the corresponding P_A = ?



$E_1 + E_P = E_2 + h_{loss}$

$$\left[z + \frac{P}{\rho g} + \frac{V^2}{2g}\right]_A + E_P = \left[z + \frac{P}{\rho g} + \frac{V^2}{2g}\right]_B + h_{loss}$$

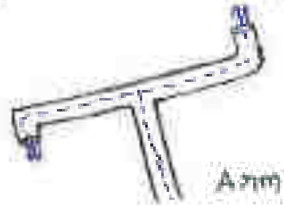
$$\frac{P_A}{\rho_w g} + 10.0 = \frac{120 \times 10^3}{\rho_w \cdot g} + 3$$

$\boxed{P_A = 51.3 \text{ kPa}}$

→ T = Moment of Momentum : (Lawn sprinkler)

$$P = \frac{2\pi N T}{60}$$

$$T = \frac{d}{dt}(mvr)$$



Axm: Horizontal plane

→ ①

$\boxed{V_1 = u_1 - R\omega}$

$\boxed{V_2 = u_2 - R\omega}$

* $\boxed{T = \dot{m}_1 V_1 R_1 + \dot{m}_2 V_2 R_2}$

$R\omega$ is in direction of ω
same dir^n of u & $R\omega$
are (+ve)

→ ②

$\boxed{V_2 = u_2 + R\omega}$

$\boxed{V_1 = u_1 - R\omega}$

* $\boxed{T = \dot{m}_1 V_1 R_1 - \dot{m}_2 V_2 R_2}$

→ ③

* $\boxed{V_3 = u_3 \sin\theta - R_3\omega}$

$\boxed{V_1 = u_1 - R_1\omega}$ $\boxed{V_2 = u_2 + R_2\omega}$

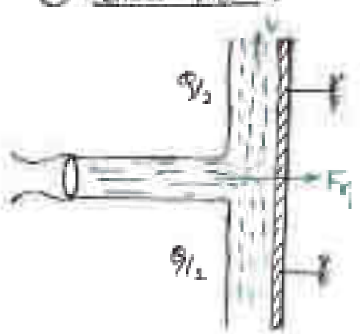
$\boxed{T = \dot{m}_1 V_1 R_1 - \dot{m}_2 V_2 R_2 + \dot{m}_3 V_3 R_3}$

★ Impact of Jet

→ linear moment equation based on Newton's second law

Ⓘ Impact of Jet on flat, smooth, vertical fixed plate / blade / vane / Bucket

① fixed plate :



$$F = ma = F\frac{(\Delta vel.)}{time}$$

$$= \frac{m}{time}(\Delta vel.)$$

$$= \dot{m}\,(\Delta vel)_n$$

$$F_n = \dot{m}(\Delta V)$$

$$\boxed{F_n = \rho Q [V_1 - V_2]_n}$$

$$P = F_n \times (\text{velocity})_{\text{blade}}$$

$$\boxed{P = F_n \times u_{(\text{blade})}}$$

$F_n = \dot m [\Delta V]_n$

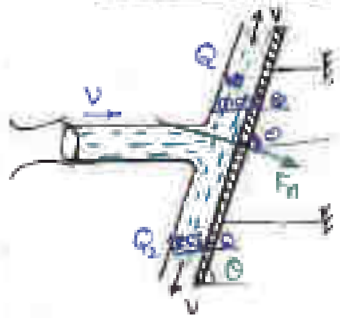
$= \rho A V [V - 0]$

$\boxed{F_n = \rho A V^2}$ for laminar $\boxed{\beta = 1.33}$

→ $P = F_n \times u$ (blade st.)

$\boxed{P = 0}$

② Inclined plate



$\boxed{Q_1 = \frac{Q}{2}[1 + \cos\theta]}$

$\boxed{Q_2 = \frac{Q}{2}[1 - \cos\theta]}$

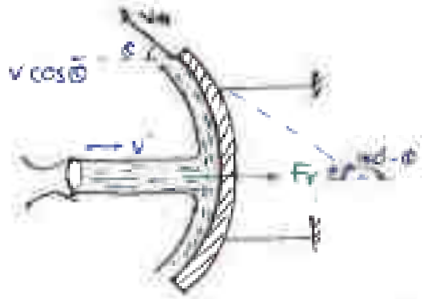
$F = \dot m (\Delta Vel)_n = \rho A V [V \sin\theta - 0]$

$\boxed{F_n = \rho A V^2 \sin\theta}$

→ $P = F_n \times u$

$\boxed{P = 0}$

③ Curved (symmetric)



$180 - \theta$ → Angle of deflection

$F_n = \dot m [\Delta Vel]_n$

$= \rho A V [V - (-V\cos\theta)]$

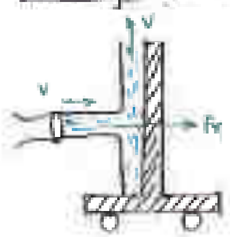
$\boxed{F_n = \rho A V^2 [1 + \cos\theta]}$

$P = F_n \times u$

$\boxed{P = 0}$

$\theta = 10 - 20^\circ$ generally → optimum Deflection angle = 160 – 170

④ Moving blade



$[V > u]$ assumption :

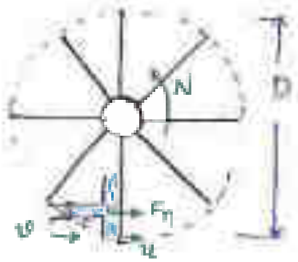
$F_n = \dot m [\Delta Vel]_n$ relative approach

$= \rho A (V - u) [V - u]$

$\boxed{F_n = \rho A (V - u)^2}$

$P = F_n \times u$

$\boxed{P = \rho A (V - u)^2 u}$



$F_n = \dot{m}\,[\Delta vel]_n$

$= \rho A V\,[V - u]$

$\boxed{F_n = \rho A V [V - u]}$

$P = F_n \times u$

$\boxed{P = \rho A V [V - u] u}$

& $\boxed{u = \frac{\pi D N}{60}}$

⇒ efficiency

$\eta = \frac{\text{output}}{\text{K.E of jet}} = \frac{\text{power}}{\text{K.E of jet}}$

$= \frac{\rho A V [V - u] u}{\frac{1}{2}\left(\frac{m}{t}\right) \cdot V^2}$ $\quad \| \; \dot{m} = \rho A V$

$$\boxed{\eta = \frac{2u(V-u)}{V^2}}$$

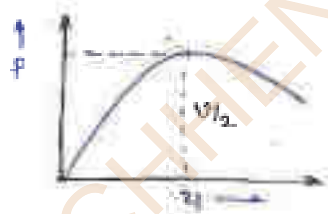
→ condition for max. efficiency

$\frac{d\eta}{du} = \frac{d}{du}\left[\frac{2u(V-u)}{V^2}\right]$

$\Rightarrow \frac{2}{V^2} \frac{d}{du}\left[Vu - u^2\right] = 0$

$V - 2u = 0$

$\boxed{u = \frac{V}{2}}$



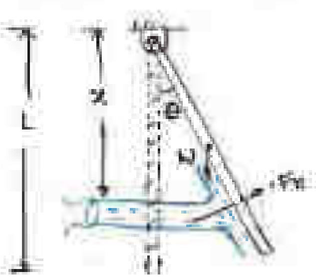
$\downarrow F_n = \rho A V [V - u]$

$= \rho A V [V - u\uparrow] u \uparrow$

$\uparrow P_1 = F_n \times u \uparrow$

⇒ special condition:

Impact of jet on Hinged plate.



$$\boxed{\sin\theta = \frac{2 \rho A V^2 x}{W L}}$$

if impact at middle of plate

$x = L/_2$

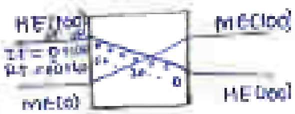
$$\boxed{\sin\theta = \frac{\rho A V^2}{W}}$$

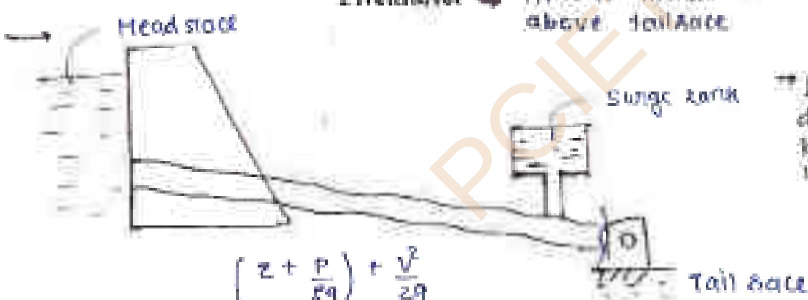
Turbine

1) Hydraulic - electric (turbine) power
2) Thermal (turbine) power
3) Nuclear power
4) Solar, wind, Gas, Bio-gas } Non conv. E.S.

→ Hydraulic electric power (turbine)

	Impulse	Reaction
Input Energy →	only K.E.	P.E + K.E.
Nozzle →	Required to convert P.E to K.E.	Not Required
Pressure across turbine →	const '0' {Patm}	Varrying, different Pressure at both sides
Casing →	simple enclosure	Leak proof casing [strong] required
Draft tube →	Not req^n	
Degree of Reaction →	D.O.R = 0	D.O.R ≠ 0; $D.O.R = \frac{R}{I+R}$
Installation →	Always install above tailrace	





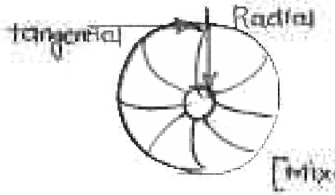
$$\left(z + \frac{P}{\rho g}\right) + \frac{V^2}{2g}$$

Hyd. E = (PE + KE)

** **Net available head**: difference in elevation betw. forebay water level & nozzle outlet

→ Turbine equation is an [illegible] representation

1) Impulse }
2) Reaction }



[Mixed flow]

Head (m)	wel	specific speed $N_s = \frac{N\sqrt{P}}{H^{5/4}}$ (N → rpm, P → kW, H → m)
- High (H > 300)	pelton wheel	Low (N_s < 60)
- Medium (60 < H < 300)	Francis / modern Francis	Medium [60 < N_s < 300]
- Low (H < 60)	Kaplan / propeller	High [300 < N_s < 1000]

PSU ★★

Pelton wheel → tangential flow I.T.
Francis turbine → Inward radial flow R.T.
Modern Francis → Mixed flow R.T.
Kaplan / propeller → Axial flow R.T.

⇒ Velocity triangle:

velocity component

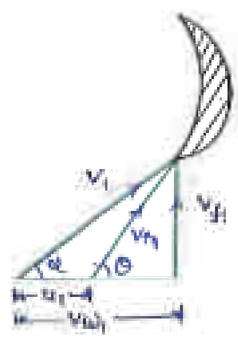
	Inlet	Exit
→ Abs velo. jet	V_1	V_2
→ Blade velocity	$u_1 = \frac{\pi D_1 N}{60}$	$u_2 = \frac{\pi D_2 N}{60}$
→ Relative velocity (tangential to blade)	V_{r_1}	V_{r_2}
→ flow velocity (flow component)	V_{f_1}	V_{f_2}
→ whirl velocity (power component)	V_{w_1}	V_{w_2}
→ Jet angle	α	β
→ Blade angle	θ	ϕ

$$\boxed{Q = A_{f_1} \times V_{f_1} = A_{f_2} \times V_{f_2}}$$

V_f → normal to area case.

$\boxed{P = \dot{m}\,[V_{\omega_1} - V_{\omega_2}]}$

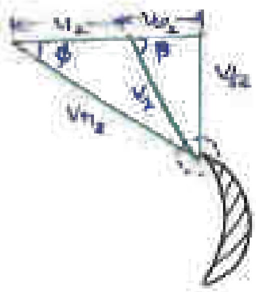
→ Inlet velocity triangle



① Pelton wheel
$\boxed{\alpha = C}$

tangential (⟶)

→ Exit velocity triangle

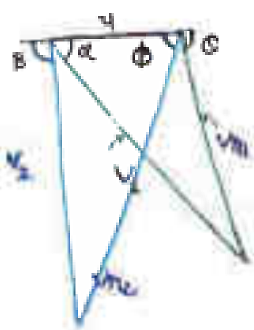


② Reaction turbine
Radial exit (β = 90°)
$V_{\omega_2} = 0$

Radial exit ($\beta = 90°$)
$\boxed{V_{\omega_2} = 0}$

→ combined velocity triangle :

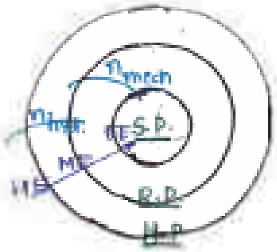
$u_1 = u_2 = u$



→ Inward Reaction turbine :- (exit at center)
- water enters the wheel at outer periphery and flow towards centre of wheel

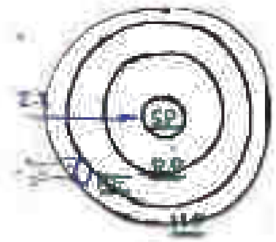
→ Outward Reaction turbine :- (exit at peripherical)
- water enters at centre of the wheel then flows towards the outer periphery of wheel

✓ Reaction turbine



→ more efficient

✓ Impulse turbine



→ less efficient
(due to more stage eg N.E)

→ S.P = shaft power = Rating

R.P = Runner power

$R.P = \rho Q\,[V_{w_1}u_1 - V_{w_2}u_2]$

$H.P = \gamma QH = \rho g QH$

$\frac{K.E \text{ of jet}}{time} = \frac{1}{2}\rho Q V_1^2$

Reaction

$\eta_{Hyd} = \frac{R.P}{H.P}$
$\eta_{mech} = \frac{S.P}{R.P}$

$\boxed{\eta_{overall} = \eta_{Hyd} \times \eta_{mech}}$

$\boxed{\eta_{over} = \frac{S.P}{H.P}}$

✓ Impulse

$\eta_{nozzle} = \frac{K.E \text{ of jet}}{H.P}$
$\eta_{Hyd} = \frac{R.P}{K.E \text{ of jet}}$
$\eta_{mech} = \frac{S.P}{R.P}$

$\boxed{\eta_{overall} = \eta_{nozzle} \times \eta_{Hyd} \times \eta_{mech}}$

⇒ **Runner power :** (R.P)

$P = F_n \times [\text{velo.}]$

$= F_n \times u$

$R.P = \rho Q\,[V_{w_1} - V_{w_2}]\,u$

$\boxed{R.P = \rho Q\,[V_{w_1}u_1 - V_{w_2}u_2]}$

→ $V_{w_1}u_1$ (+ve)

$V_{w_2}u_2$ (−ve) or zero

	−ve	zero
u_2	×	×
V_{w_2}	✓ only IT × RT	✓ RT

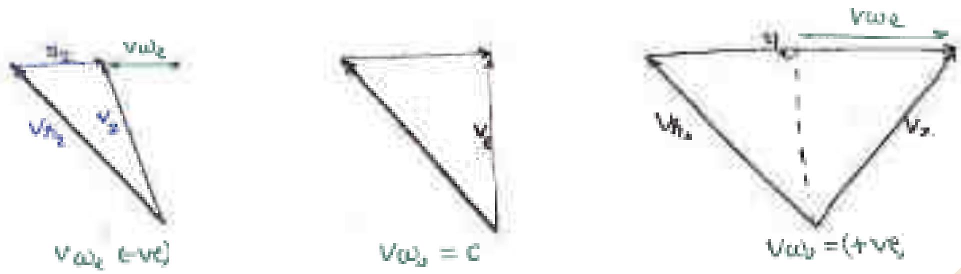
NOTE: To have ... negative for impulse turbine as it is not possible for reaction turbine ∴ radial discharge is preferable.
($\because V_{w_2} = 0$)

$$R.P_{I.T} = \rho Q\left[V_{w_1}u_1 + V_{w_2}u_2\right]$$

$$R.P_{R.T} = \rho Q\left[V_{w_1}u_1 - V_{w_2}u_2\right]$$

$$= \rho Q\left[V_{w_1}u_1\right]$$

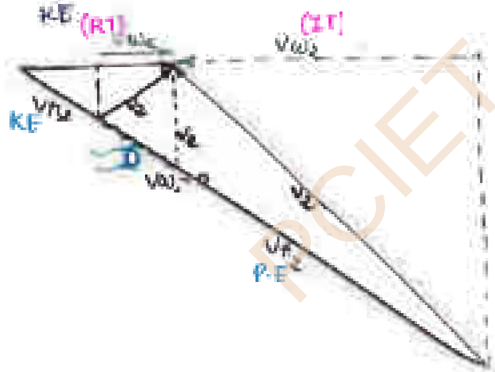
[$\because V_{w_2} = 0$ radial exit]



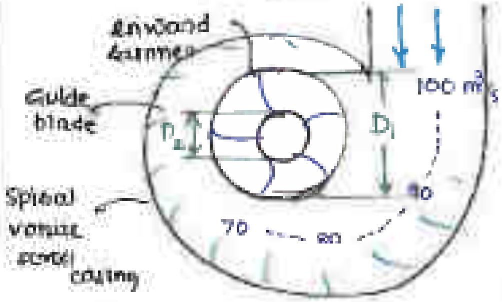
I.T ✓ (I.T) ✓ ✓

R.T ✓ (R.T) ✓

$$z + \underbrace{\frac{p}{\rho g}}_{\substack{60 \\ PE}} + \underbrace{\frac{V^2}{2g}}_{\substack{40 \\ KE}} = 100$$



Inward Radial flow R.T



$$\boxed{\begin{array}{l} u_1 = \frac{\pi D_1 N_1}{60} \\ u_2 = \frac{\pi D_2 N_2}{60} \end{array}}$$

$\downarrow Q = \downarrow A\ V$ (const)

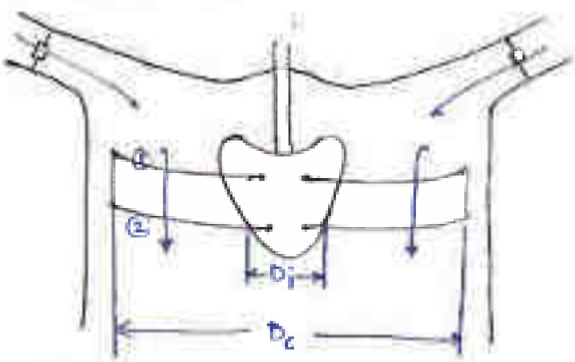
to main^n const. velocity of runner vane. Velocity not allow create vibration

→ **NOTE**: Why spiral volute casing is used is:

– To maintain const velocity of water circulating around runner the flow area will required to decrease in the same proportion of flow rate reduction

→ **Kaplan / propeller turbine**

Axial flow runner turbine:



$$\boxed{u_1 = u_2 = \frac{\pi D N}{60}}$$

where $D = \frac{D_o + D_i}{2} = D_{mean}$

– It kept at vertical

– For axial flow

$Q = \pi D_1 B_1 V_{f1} = \pi D_2 B_2 V_{f2}$

area of flow const

$$\boxed{V_{f1} = V_{f2}}$$

→ If blades are.

i) permently fixed → propeller (Rigid)

ii) Adjustable → Kaplan

PSU

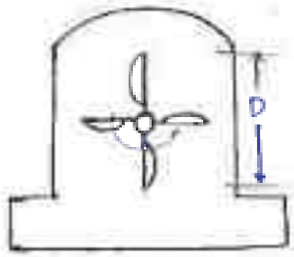
$\eta_{Hyd} = \frac{R.P}{H.P.} = \frac{\rho Q [V_{w1} u_1]}{\rho Q g H}$

**

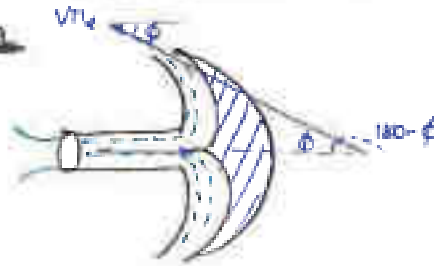
$$\boxed{\eta_{Hyd\ R.T} = \frac{V_{w1} u_1}{gH}}$$

***

[Tangential flow I.T]

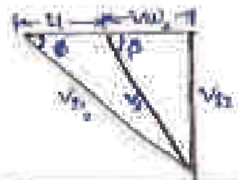


splitter



$\boxed{V_1 = V_{w_1}}$

$\boxed{V_{r_1} = V_1 - u}$



$\boxed{V_{r_2}\cos\phi = u + V_{w_2}}$

I.T (pelton wheel)

$S.P = \text{Rating}$

$\boxed{R.P. = \rho Q\,[V_{w_1} + V_{w_2}]\,u} \leftarrow I.P$

$K.E_{jet} = \frac{1}{2}\rho Q V_1^2$

$H.P = \rho Q g H$

$\boxed{T = \rho Q\,[V_{w_1} + V_{w_2}]\,R}$

⇒ efficiency for I.T :

1) $\eta_{nozzle} = \dfrac{K.E \text{ of jet}}{H.P} = \dfrac{\frac{1}{2}\rho Q V_1^2}{\rho g Q H}$

$= \dfrac{\frac{1}{2}\rho Q(\sqrt{2gH})^2 C_v^2}{\rho g Q H}$

{ $V_1 = C_v\sqrt{2gH}$ valid for I.T only

$\boxed{\eta_{nozzle} = C_v^2}$ ← speed ratio

2) $\eta_{hyd} = \dfrac{R.P}{K.E \text{ jet}} = \dfrac{\rho Q\,[V_{w_1} + V_{w_2}]\,u}{\frac{1}{2}\rho Q V_1^2}$

$\boxed{\eta_{Hyd} = \dfrac{2u\,[V_{w_1} + V_{w_2}]}{V_1^2}}$ ← I.T

$$\boxed{\eta_{Hyd_{max}} = \frac{1 + K\cos\phi}{2}}$$; where ϕ = Bucket exit angle

→ optimum deflection angle → 160° to 170°

→ 3) $$\boxed{\eta_{mech} = \frac{S.P}{R.P}}$$

4) overall efficiency.

$$\boxed{\eta_o = \eta_{nozzle} \times \eta_{Hyd} \times \eta_{mech}}$$

How to solve questions ??

1) $V_1 = V$ (given)

$= C_V\sqrt{2gH}$

$= \left[\frac{Q}{\pi/4\, d^2}\right]$

2) $u = V$

$= \frac{\pi D N}{60}$, (RW)

= optimum cond^n = $V_1/2$

= speed ratio (G) = $\left(\frac{u}{\sqrt{2gH}}\right)$

⟹ Jet ratio (m)

$$\boxed{\text{Jet ratio (m)} = \frac{D}{d} = \frac{\text{Dia of runner}}{\text{Dia of jet}}}$$

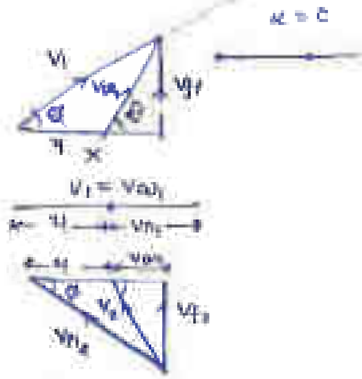
→ $$\boxed{\text{No of blades} = 15 + \frac{D}{2d} = 15 + \frac{m}{2}}$$

	No of blades	Jet ratio
pelton wheel	18 – 25	11 – 14
francis	12 – 16	
Kaplan	4 – 6	

→ Speed ratio of pelton wheels vary from 0.43 to 0.47

Francis turbine vary from 0.6 to 0.9

GATE-06 water jet velocity at entry = 25 m/s with a flow rate of 0.1 m³/s ; deflection angle of 120° ; assuming an ideal flow power develop by runner in (kW) = (?)



→ $u = 10$ m/s
$V_1 = 25$ m/s

→ $R.P = \rho Q [V_{w_1} + V_{w_2}] u$

angle of deflection $= 180 - \phi$
$\phi = 60$

$V_1 = V_{w_1}$ — (i)

$V_{w_2} = V_{r_2} \cos\phi - u$ — (ii)

∴ $V_1 = V_{w_1} = 25$

$V_{r_1} = 25 - 10 = 15$

$V_{r_2} = K V_{r_1} = 15$ m/s

∴ $V_{w_2} = 15 \cos 60 - 10$
$= -2.5$ m/s

→ $R.P = \rho Q [V_{w_1} + V_{w_2}] u = 1000(0.1)(25 + (-2.5))10$

$\boxed{R.P = 22.5 \text{ kW}}$

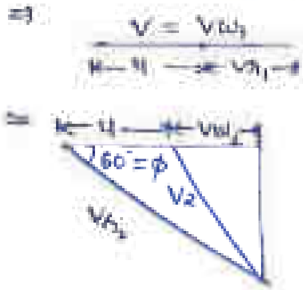
GATE 2008 water issues out of a nozzle with velocity of 10 m/s & strikes on to the bucket of the pelton wheel rotating at 10 rad/s . The mean dia. of 1 m & jet gets deflected by 120° neglecting the blade friction : The torque devel. per unit mass flow rate (?)

⇒ $u = \frac{\pi \times D \times N}{60} = \omega \times R$

$= 10 \times 0.5$

$\boxed{u = 5 \text{ m/s}}$ & $\boxed{V_1 = 10 \text{ m/s}} = V_{w_1}$

so, $V_{r_1} = 10 - 5 = 5$ m/s



→ $V_{r_1} = V_{r_2} = 5$ m/s frictionless

$V_{r_2} \cos\phi = u + V_{w_2}$

$(5) \cos 60 = 5 + V_{w_2}$

$\boxed{V_{w_2} = -2.5 \text{ m/s}}$

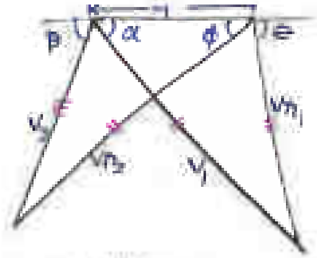
per unit mass given ($T/\rho Q$ = ?)

→ $T = (\rho Q (V_{w_1} + V_{w_2}) \cdot R$

$= (10 - 2.5)(0.5)$

$\boxed{T = 3.75 \text{ N/m}}$ Ans

GATE -12



$V_1 = V_{r_2}$ | $\alpha = \phi$

$V_{r_1} = V_2$ | $\beta = \theta$

Degree of reaction = D.R (?)

$$D.O.R = \frac{(\Delta H)_{rotor}}{(\Delta H)_{stage}}$$

$$D.O.R = \frac{(\Delta H)_{rotor}}{(\Delta H)_{stator} + (\Delta H)_{rotor}}$$

$$D.O.R = \frac{(\Delta H)_{rotor}}{(PE_R + KE_T)}$$

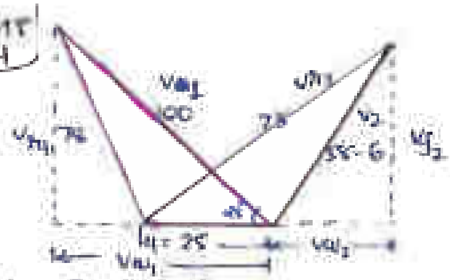
$$D.O.R = \frac{PE_R}{PE_R + KE_T}$$

$$= \frac{[\Delta P]_m}{(RP)_{wall}} = \frac{\frac{(P_1 - P_2)}{\rho g}}{(R.P)_{per\ wt}}$$

$$D.O.R = \frac{\frac{P_1 - P_2}{\rho g}}{\frac{\rho Q (V_{w_1} u_1)}{\rho g Q}} \Rightarrow \boxed{D.O.R = \frac{\left(\frac{P_1 - P_2}{\rho g}\right)}{V_{w_1} u_1}}$$

(case of Francis turbine)

GATE -14



Axial flow impulse turbine

Jet velocity = 100 m/s

$\alpha = 25°$

Blade velocity = 25 m/s

$V_{r_1} = V_2$, $V_{f_1} = V_{f_2}$

Specific work in J/kg —

$$\rightarrow P = \rho Q [V_{w_1} u_1 + V_{w_2} u_2]$$

$$= \rho Q [V_{w_1} + V_{w_2}] u \quad W$$

$$W = J/s = J/kg \cdot kg/s = P$$

$$J/kg = \frac{\rho Q [V_{w_1} + V_{w_2}] u}{m} = [V_{w_1} + V_{w_2}] u$$

$$\rightarrow \cancel{V_{r_1} + u = V_1 \cos\alpha}$$

$$\cancel{\boxed{V_{r_1} = 65 \text{ m/s}}}$$

$$V_{w_1} = 100 \cos 25°$$

$$\boxed{V_{w_1} = 90.63 \text{ m/s}}$$

$$\Rightarrow V_{f_1}^2 + (25 + 65)^2 = 100^2$$

$$\boxed{V_{f_1} = 43.58 \text{ m/s}} = V_{f_2}$$

$$V_2^2 = V_{f_1}^2 + V_{w_2}^2$$

$$(58.6)^2 - (43.58)^2 = V_{w_2}^2$$

$$\boxed{V_{w_2} = 39.17 \text{ m/s}}$$

power = $(V_{w_1} + V_{w_2}) u$ = 3245 J/kg

ESE The discharge in the penstock is 1 m^3/s at an efficiency of the $\eta = 80\%$ can develop a power of (shaft) =

→ If nothing mentioned efficiency take overall efficiency

→ If types of turbine not mentioned take reaction turbine

$$\eta_{over} = \frac{S.P}{H.P}$$

$$S.P = \eta_{over} \times \gamma Q H$$

$$= 0.8 \times 9810 \times 1.0 \times 100$$

$$\boxed{S.P = 784.8\ kW}$$

→ If $\eta_{stage} = 80\%$ given & $\eta_{overall} = 80\%$?

$$\eta_{overall} = \left(\frac{S.P}{R.P}\right)\left(\frac{R.P}{H.P}\right) = \frac{S.P}{H.P}$$

$$S.P = \eta_{Hyd} \times \eta_{mech} \times \gamma Q H$$

$$= 0.8 \times 0.8 \times 9810 \times 1.0 \times 100$$

$$\boxed{S.P = 627.8\ kW}$$

→ If $\eta_{nozzle} = 80\%$ given then

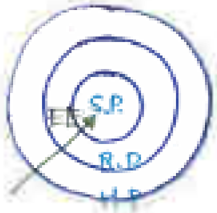
$$\eta_{O} = \eta_{nozzle} \times \eta_{Hyd} \times \eta_{mech} = \frac{S.P}{H.P}$$

$$S.P = \eta_{nozzle} \times \eta_{Hyd} \times \eta_{mech} \times \gamma Q H$$

$$= 0.8 \times 0.8 \times 0.8 \times 9810 \times 1.0 \times 100$$

$$\boxed{S.P = 502\ kW}$$

→ Francis turbine ⇔ centrifugal pump (working by Francis rush)



S.P = Rating

$[\leftarrow R.P]_{FT} = I.P = \rho Q\,[V_{w_2}u_2 - V_{w_1}u_1]$

for radial entry $V_{w_1} = 0$ (no whirl generation)

$$\boxed{\text{Manometric power} = M.P. = \gamma Q H_m}$$

→ $\eta_{mech} = \dfrac{I.P.}{S.P.}$

$\eta_{mano} = \dfrac{M.P}{I.P}$

$\eta_{vol^m} = \dfrac{Q - \Delta Q}{Q}$

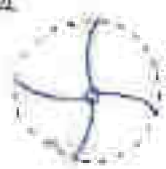
→ $\eta_{overall} = \eta_{mano} \times \eta_{mech} \times \eta_{vol^m}$

→ $$\boxed{\eta_{Hyd\,(H.T)} = \frac{V_{w_1}u_1}{gH}}$$

→ $$\boxed{\eta_{mano} = \frac{\gamma Q H}{\rho Q\,[V_{w_2}u_2\;]}}$$ *

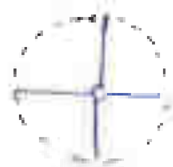
→ $$\boxed{\eta_{mano} = \frac{g H_m}{V_{w_2}u_2}}$$

Impeller



Forward Blading

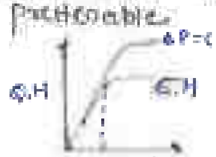
Backward Blading



Radial Blading

NOTE

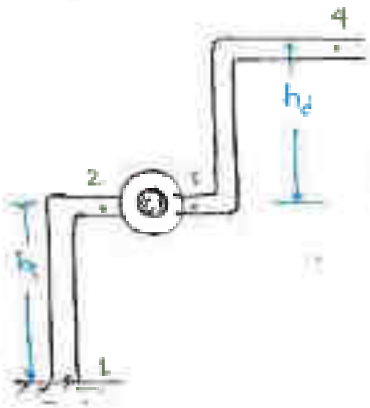
- Though the discharge & Head developed is more for forward blading, backward blading is preferable.
  - Due to surging / stalling



Forward

Manometric Head } H_m
Gallune Head

$$\boxed{H_m = h_s + h_d + h_{fs} + h_{fd}}$$



→ In priming
$h_s \downarrow$; $h_s \uparrow$
$h_d \downarrow$; $h_d \uparrow$
suction Head decreases
→ to avoid cavitation
→ $P_{min} > P_{vapour} \Rightarrow \sigma > \sigma_c$

Here ; σ = thoma Number = $\frac{NPSH}{H}$

σ_c = critical σ
= cavitation coefficient

$\sigma = \frac{NPSH}{H} > \sigma_c$

$$\boxed{\frac{NPSH}{H} > \sigma_c}$$

→ $NPSH = \frac{P_{atm}}{\rho g} - h_s - h_{fs} - \frac{P_{vapour}}{\rho g}$

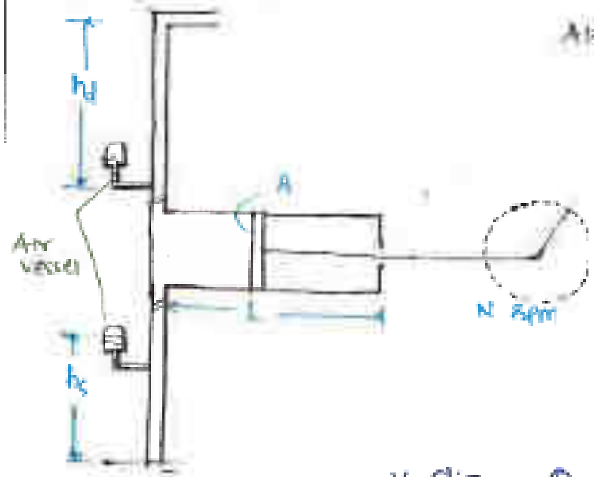
$$\boxed{NPSH = \frac{P_{atm}}{\rho g} - \left[h_s + h_{fs} + \frac{P_{vapour}}{\rho g}\right]}$$

→ Net positive [suction]

E

$$\begin{array}{ccc} & A \rightarrow 20 & \\ & \downarrow & \} \text{ 1C unit} \\ & B \rightarrow 10 & \\ & \downarrow & \} \text{ 1C} \\ 2C & C \rightarrow 0 & \\ \uparrow & \downarrow & \\ 1C & D \rightarrow -1C & \end{array}$$

D unit (+ve)



Air vessel ⇒ It required to avoid the Hammering & damping effect

$Q = m^3/sec$

$= \left(\frac{Vol^m}{stroke}\right)\left(\frac{stroke}{Rev}\right)\left(\frac{Rev}{min}\right) / 60$

$$\boxed{Q_{th} = \frac{ALN}{60} \ \frac{m^3}{sec.}}$$

$$\% \text{ Slip} = \frac{Q_{th} - Q_{act}}{Q_{th}} = 1 - \frac{Q_{act}}{Q_{th}}$$

$$\boxed{\% \text{ slip} = 1 - \frac{Q_{act}}{Q_{th}} = 1 - C_d}$$

NOTE

⇒ −ve slip $[Q_{act} > Q_{th}]$ is possible when
(i) pump running at high speed
(ii) $h_s >>> h_d$

II

1st (1) unit Quantity

$\boxed{N_u = \frac{N}{\sqrt{H}}}$; $\boxed{Q_u = \frac{Q}{\sqrt{H}}}$; $\boxed{P_u = \frac{P}{H^{3/2}}}$

2nd (2) Model studies

$\boxed{\frac{\sqrt{H}}{DN} = C}$; $\boxed{\frac{Q}{D^3 N} = C}$; $\boxed{\frac{P}{D^5 N^3} = C}$

3rd (3) Specific speed (Ns)

{Turbine}

N_s $\boxed{\frac{N\sqrt{P}}{H^{5/4}}}$

{Pump}

N_s $\boxed{\frac{N\sqrt{Q}}{H^{3/4}}}$

GATE 08 | ... centrifugal pump ... [illegible]
Req^d to drive the pump

→ $N_1 \to N$ & $N_2 \to 2N$

$$\boxed{\frac{P}{D^5N^3} = C}$$ model studies

$$P_1 = CD^5N_1^3$$

$P \propto N^3$ → $P_1 \to N_1$ & $P_2 = (2)^3 N_1$

$$\boxed{P_2 = 8P_1}$$ 8 times increase

follow : 1) Dim^n Geometric similarities
2) Kinematic similarity
3) Dynamic similarity

GATE 05 | A model of a turbine is working Head of $1/4^{th}$ of that under which the full scale turbine works. & the dia. of model is ½ of full scale. & N is Rpm of model. RPM of full scale will be :

→

	F S	model
Head	H	$H/4$
Dia	D	$D/2$
RPM	N_1	N

from model studies

$$\left[\frac{\sqrt{H}}{DN}\right] = \left[\frac{\sqrt{H}}{DN}\right]_{model}$$

$$\frac{\sqrt{H}}{DN_1} = \frac{\sqrt{H/4}}{D/2\, N}$$

$$\boxed{N_1 = N}$$

Q-10) Turbine working a Head of 40 m was developing power of 1000 kW ; if Head reduced to 20. the

$$\left[\frac{P}{H^{3/2}}\right]_1 = \left[\frac{P}{H^{3/2}}\right]_2$$ ⇒ unit qty

$$1000\left(\frac{20}{40}\right)^{3/2} = P$$

$$\boxed{P = 353.3 \text{ kW}}$$

developing power of 3000 kW while running 1000 rpm for entire testing a 1 : 4 scale model of turbine working under head of 10 m can develop power = (?)

H, P, N

$V = \frac{\pi D N}{60}$

$\frac{P_1}{D_1^5 N_1^3} = \frac{P_2}{D_2^5 N_2^3}$

$P_2 = 3000$

$\frac{\sqrt{H}}{DN} = \frac{P}{D^5 N^3} = C$

$D = \frac{\sqrt{H}}{CN}$

$\frac{P_1}{\left(\frac{\sqrt{H_1}}{CN_1}\right)^5 N_1^3} = C = \frac{P_2}{\left(\frac{\sqrt{H_2}}{CN_2}\right)^5 N_2^3}$

$N_s = \frac{N}{\sqrt{H}}$

$\frac{N_1}{N_2} = \sqrt{\frac{H_1}{H_2}} = \sqrt{\frac{40}{10}}$

$1000 = 2 \times N$

$\boxed{N_2 = 500}$

$\frac{3000}{\frac{(\sqrt{40})^5 (1000)^3}{(1000)^5 C^5}} = \frac{P_2}{\frac{(\sqrt{10})^5 (500)^3}{(500)^5 C^5}}$

$\frac{(3000)(1000)^2}{(40)^{5/2}} = \frac{P_2 (500)^2}{(10)^{5/2}}$

$P_2 = 2371(816.22)$

$\boxed{P_2 = 2.34\ kW}$

$\longrightarrow$ $\frac{\sqrt{H}}{DN} = C$; $\frac{P}{D^5 N^3} = C$

$\rightarrow$ $D_{full\,scale} = 4 D_{model}$

$\frac{P}{D^5 (DN)^3} \Rightarrow \frac{P}{D^2 (\sqrt{H})^3}$

$\left[\frac{P}{D^2 H^{3/2}}\right]_{ft} = \left[\frac{P}{D^2 H^{3/2}}\right]_{model}$

scale → 1 : 4

$\frac{300}{(4D)^2 \times 40^{3/2}} = \frac{P_2}{(D)^2 \times 10^{3/2}}$

$\boxed{P_2 = 2.34\ kW}$

Ex

Atm. pr. Head = 10.5 m $\left(\frac{P}{\rho g}\right)$

Suction Head = 40 m (H)

Vapour pr Head = 2.5 m $\left(\frac{P_v}{\rho g}\right)$

Cavitation coefficient = 0.15 (σ_c)

— The max height at which turbo m/c (h_s) can set above tail race level = ?

Ans

$$P_{min} > P_{vapour}$$

$$\frac{NPSH}{H} > \sigma_c$$

$$\frac{P_{atm}}{\rho g} - \left[h_s + h_{f_s} + \frac{P_{vap}}{\rho g}\right] > \sigma_c H \qquad (h_{f_s} \to 0)$$

$$10.5 - h_s - 2.5 > 0.15 \times 40$$

$$\boxed{h_s < 2\,m}$$

Run away speed :-

- Run away speed is a maximum speed that a turbine can experience under no load condition with wicket gate wide open. (Max. mass flow rate)

Draft tube :

- Draft tube is used at exit of an reaction turbine to conve large portion in K.E going of waste into useful pressure energy
- Hence make it possible to erect the turbomachinery above turbo tail race level

$$Q = A \uparrow V \downarrow$$

$$z + \frac{P}{\rho g} + \frac{V^2}{2g} = C$$

$$KE \to PE$$

Extend in syllabus for state PSU's

- pump ✓
- turbo performance curves
- Hyd devices → Hyd press, RAM → etc.

— Dimensional formula

Mass	Lenght	Time	temp
M	L	T	θ

Eg

velocity $= L^1 T^{-1}$

Accl^n $= L^1 T^{-2}$

ω = rad/s $= T^{-1}$

$F = ma = M^1 L^1 T^{-2}$

— ① Reyleigh's Method

② Buckingham's 'π' theorem

Where : m = no. of total variable

η = no. of repeted variable

No. of π - terms Dimensionless terms $= m - \eta$

Ex $Re = \frac{\rho V L}{\mu}$ → ρ, V, L, μ, Re

m = total no. of variable = 5

η = no. of repeted varia

Selection of repeted variable

① At least one should present

i) Geometry property

ii) fluid property

iii) flow property

② selected group should be dimensional

③ Must fundamental (least dimensions)

$\rho = M^1 L^{-3}$ ← (ρ, μ) both fluid

$V = L^1 T^{-1}$ ← ~~fluid~~ flow property

$L = L$ ← Geometry property

$\mu = M^1 L^1 T^{-1}$

$Re = M^0 L^0 T^0$

π terms $= m - \eta = 5 - 3 = 2$

— π_1 & π_2

$\pi_2 = [\rho^{a} V^{b} L^{c}]\, Re$

Here $\pi_1 = \rho^{a_1} V^{b_1} L^{c_1} \cdot \mu$

$(M^0 L^0 T^0) = (ML^{-3})^{a_1} \cdot (LT^{-1})^{b_1} (L)^{c_1} \cdot (M^1 L^{-1} T^{-1})$

power of 'M' → $0 = a_1 + 1$ → $\boxed{a_1 = -1}$

'L' → $0 = -3a_1 + b_1 + c_1 - 1$ → $0 = 3 + b_1 + c_1 - 1$

$b_1 + c_1 = -2$

'T' → $0 = -b_1 - 1$ → $\boxed{b_1 = -1}$ → $\boxed{b_1 = -1}$

$\pi_1 = [\rho^{-1} V^{-1} L^{-1}]\, \mu$

$$\boxed{\pi_1 = \frac{\mu}{\rho V L}}$$

→ $\boxed{\pi_2 = Re}$

but $\pi_2 = f(\pi_1)$

so ; each π - term containg = $n+1$ variable for Ex (4)

⇒ Dimensionless Number :

1) Reynolds NO. = $\frac{\text{Interial force}}{\text{viscous force}} = \frac{F_i}{F_v} = \frac{\rho V L}{\mu}$

2) Froude NO = $F_r = \sqrt{\frac{\text{Intertia force}}{\text{gravity force}}} = \sqrt{\frac{F_i}{F_g}} = \frac{V}{\sqrt{gL}}$

$$\boxed{F_r = \frac{V}{\sqrt{gL}}}$$

3) Weber NO = $We = \sqrt{\frac{\text{Interia force}}{\text{Surface tension}}} = \sqrt{\frac{F_i}{F_\sigma}} = \frac{V}{\sqrt{\sigma/\rho L}}$

$$\boxed{We = \frac{V}{\sqrt{\sigma/\rho L}}}$$

4) Mach NO = $M = \sqrt{\frac{\text{Interia force}}{\text{elastic force}}} = \frac{V}{\sqrt{K/\rho}}$

$$\boxed{M = \frac{V}{\sqrt{K/\rho}} = \frac{V}{C}}$$ obj

$\boxed{\gamma RT}$

sound

$C = \sqrt{K/\rho} = \gamma RT$

$\sqrt{\frac{}{\text{pressure force}}} = \frac{V}{\sqrt{P/\rho}}$

$$\boxed{E_u = \frac{V}{\sqrt{P/\rho}}}$$

Newton's No = Newt $= \frac{1}{E_u} = \frac{\sqrt{P/\rho}}{V}$

$$\boxed{\text{New.} = \frac{\sqrt{P/\rho}}{V}}$$

pressure coefficient $= \left(\frac{1}{E_u}\right)^2 = \frac{P}{\rho V^2}$

⇒ classification :

$M << 1$ → ultrasonic

$M < 0.4$ → compressibility effect can be neglected

$0.9 < M < 1.1$ → transonic

$M < 1$ → subsonic

$M = 1$ → sonic

$M > 1$ → supersonic

$M > 5$ → hypersonic

PSQ

$F_r = 1$ critical flow

$F_r < 1$ sub critical [Tranquil flow]

$F_r > 1$ super critical [Torrential flow]

Q-2007] $Re = 5$; $\frac{F_i}{F_v} = (5) = 5$

Q-14] For a flow water from circular pipe of 100 m dia with velocity of 0.1 m/s rate of 36 kg/hr & viscosity = 0.001 kg/m-s Re = (?)

$Re = \frac{\rho V D}{\mu} = \frac{kg/m^3 \cdot m/s \cdot (100)}{0.001}$

$= \frac{(0.01)(100)}{(1000)(0.001)}$

$\dot{m} = 36\ kg/hr = 36/3600\ kg/s = \rho Q$

$V = \frac{Q}{\pi/4\, d^2}$; $Re = \rho\left(\frac{Q}{\pi/4\, d^2}\right) \cdot \frac{d}{\mu}$

$= \frac{4 \dot{m}}{\pi d \mu} = \frac{4 \times 36}{\pi \times 3600 \times 10^{-3} \times 0.1} = 127$

$\boxed{Re = 127}$

GATE-16 $\left(P + \frac{a}{v^2}\right)(v - b) = R$

$$P + \frac{a}{v^2} = \frac{RT}{(v-b)}$$

$$= \frac{\frac{kJ}{kg\,K}\,K}{\frac{m^3}{kg}}$$

$$\frac{a}{m^3 / v^2} = \frac{kJ}{kg \cdot m^3}$$

$$a = \frac{kg \cdot m/s^2 \cdot m}{kg \cdot m^3 \cdot m^3}$$

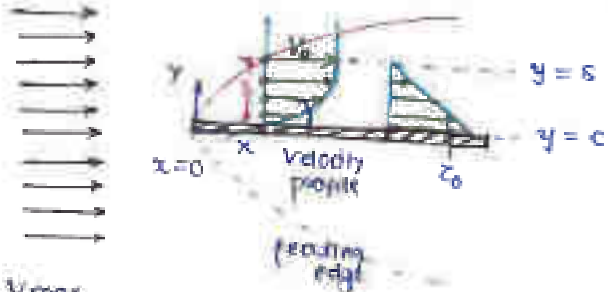
$$\boxed{v = \text{specific vol}^m = \frac{m^3}{kg}}$$

$$\text{Dim } P = \text{Dim}\left(\frac{a}{v^2}\right)$$

$$\frac{N}{m^2} = \frac{a}{\left(\frac{m^3}{kg}\right)^2}$$

$$\boxed{a = \frac{m^5}{kg\,sec^2}}$$

→ Boundary layer theory is given by L. Prandtl (in 1904)



$V_0 = V_{max}$
= free stream velocity

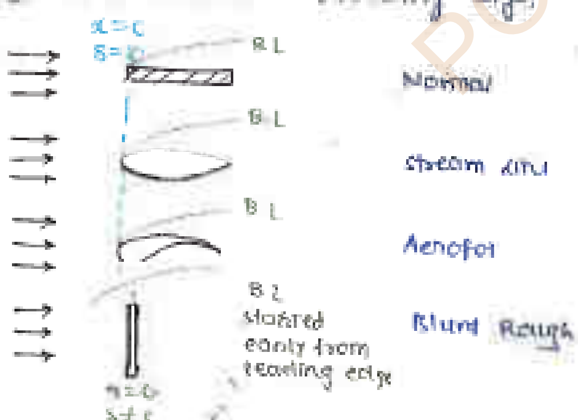
condition for "NO SLIP"

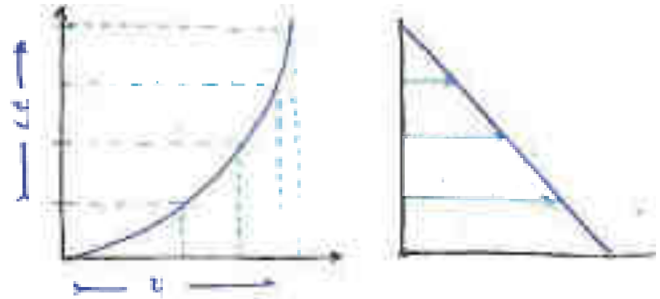
- Whenever real fluid flows over solid boundary because of No slip condition fluid particles will get stick to the boundary ; Hence velocity of particles will be equal to velocity of boundary if the object is at rest. fluid particle velocity near the boundary will be zero and at a greater distance in normal dir^n particle velocity keeps on increasing at reaching at max. value at a dis^n of δ known as boundary layer thickness ; This zone where velocity gradient exist is the boundary layer zone.

⇒ Boundary conditions :

(1) External flow :

① $y = 0$ → at boundary ; $V = 0$
② $y = \delta$ → $V = V_0 = V_{max}$
③ $y = 0$ → $\tau = \tau_0 = \tau_{max}$ [∵ $\frac{du}{dy}$ is max]
④ $y = \delta$ → $\tau = \tau_{min} = 0$
⑤ $x = 0$; $\delta = 0$ (Leading edge)





→ As distance increases from the leading edge phase transform scale is increases so the potential difference will also increase & driving potential is max at $y = \delta$

Ex.

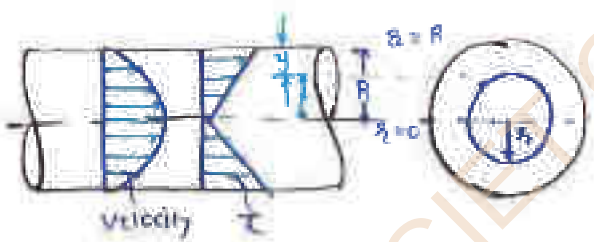
		time
100°C	}ΔT	2 sec
90°C	}ΔT	5 sec
80°C		
40°C	}ΔT	5 hr
30°C		

However same ΔT dist^n the time req^n to reaching the 90°C to 80°C is more because due to phase transform time requires more.

Same: In welding nearer to surface grain size is more fine compare to outer side.

② Internal flow

→ flow through pipe

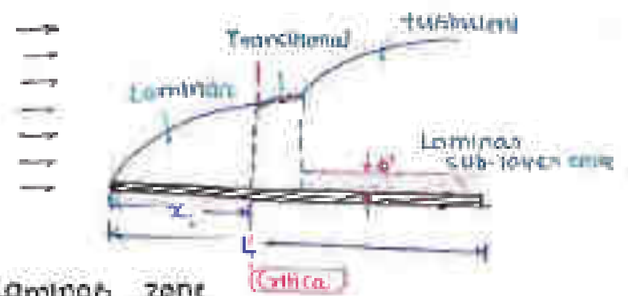


$r = 0$, center ; $V = V_0 = V_{max}$; $\tau = \tau_{min} = 0$

$r = R$; wall ; $V = 0$; $\tau = \tau_{max} = \tau_w$

$y = R - r \Rightarrow dy = -dr$

$$\boxed{\tau = \mu\left(-\frac{du}{dy}\right)}$$



① Laminar zone

- The velocity profile with a laminar zone is parabolic & is given by

$$\boxed{\frac{V}{V_0} = \left(\frac{y}{\delta}\right)^{\eta} \; ; \; \eta = 1 \text{ to } 3}$$ (parabolic)

if nothing is mentioned take $\eta = 1$

$$\boxed{\frac{V}{V_0} = \left(\frac{y}{\delta}\right)}$$ (Nothing –

② Turbulent zone :

- velocity profile is the logarithmic & is is given by

$$\boxed{\frac{V}{V_0} = \left(\frac{y}{\delta}\right)^{\eta} \; ; \; \eta = 1/7}$$ (logarithmic)

if nothing is mentioned take $\eta = 1/7$

$$\boxed{\frac{V}{V_0} = \left(\frac{y}{\delta}\right)^{1/7}}$$

- Governed by $1/7^{th}$ power law

⟹ Laminar sub layer zone (δ')

- Within the turbulent zone near the boundary always laminar sub layer will exist though the actual profile is parabolic it will be treated as linear on practical calculation.

$$\boxed{\delta' = \frac{11.6\,\nu}{V_*}}$$

where : ν = kinematic velocity

V_* = shear velocity = $\frac{\tau_0}{\rho}$

$$\boxed{\delta' \propto \frac{1}{Re}}$$

→ Limiting condition to be in laminar zone [length of laminar zone] will be given by critical condition

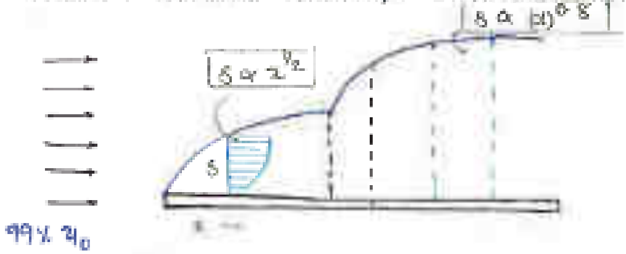
$\boxed{Re_{local} < Re_{critical}}$ $\quad Re_{critical} = 5\times10^5$

$\boxed{Re_x < 5\times10^5}$ (for flat plate)

$\boxed{Re_x < 2000}$ (for circular pipe)

→ **Boundary layer thickness (δ)**

→ It is dist^n from the boundary to a point in normal dir^n where velocity reaches 99% its maximum velocity.



$\boxed{\dfrac{\delta}{x} = \dfrac{5}{\sqrt{Re_x}}}$ (laminar) → $\boxed{\delta \propto \sqrt{x}}$

$\boxed{\dfrac{\delta}{x} = \dfrac{0.376}{(Re_x)^{1/5}}}$ (turbulent) → $\boxed{\delta \propto (x)^{0.8}}$

NOTE :

→ above eq^n based on Blasius's experimental result which can be used in absence of actual velocity profile

→ when actual velocity profile is known Von Karman's momentum integral

$$\boxed{\frac{\tau_0}{\rho V_0^2} = \frac{d\theta}{dx}}$$

where θ = momentum thickness

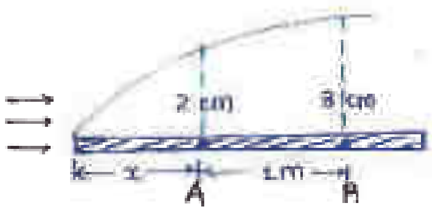
$$\boxed{\theta = \int_0^{\delta} \frac{V}{V_0}\left[1 - \frac{V}{V_0}\right] dy}$$

GATE 2001) The laminar boundary layer thickness at a point 'A' on a flat plate at point 'A' is 2 cm. and at point is 1 m downstream of 'A' is 3 cm. the dist^n of 'A' from leading edge of the plate

→ $\delta_A = 2$ cm → $x_A = 3$ cm
$\delta_B = 100$ cm → $x_B = ?$

$$\frac{\delta}{\sqrt{x}} = \text{const} \Rightarrow \frac{\delta_A}{\sqrt{x_A}} = \frac{\delta_B}{\sqrt{x_B}}$$

$$x_B = \left(\frac{\delta_B}{\delta_A}\right)^2 x_A = \left(\frac{100}{2}\right)^2 (3) = \frac{10000}{4} \times 3$$



	A	B
δ	2	3
dist^n	x	x+1

$$\frac{\delta_1}{\delta_2} = \sqrt{\frac{x_1}{x_2}} = \sqrt{\frac{x}{x+1}} = \frac{2}{3}$$

$$\frac{x}{x+1} = \frac{4}{9}$$

$$\boxed{x = 0.8}$$

Ex For flow of an air stream over a flat plate of 1.2 m wide and 2 m long with free stream velocity of 6 m/s $\rho = 1.2$ kg/m^3 ; $\nu = 1.47 \times 10^{-5}$ m^2/s upto what length over the plate will boundary layer be laminar

→ $Re = \frac{\rho V L}{\mu} = 5 \times 10^5 = R_{critical}$

$$\boxed{Re_{local} < Re_{critical}}$$

$$\frac{6 \times L}{1.47 \times 10^{-5}} \leq 5 \times 10^5$$

$$\boxed{L \leq 1.225 \text{ m}}$$

Q-R) For a flow over a flat plate ; at $R_L = 1000$ (laminar) at an instant the boundary layer thickness 4 mm is the velocity alone is increased by 4 factor of 4 δ at that location in mm

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}} \qquad Re = \frac{\rho V D}{\mu} \rightarrow Re_2 = 4 Re$$

$$\delta_1 \sqrt{Re_1} = \delta_2 \sqrt{Re_2}$$

$$\delta_2 = \frac{4 \times \sqrt{Re}}{2\sqrt{Re}}$$

$$\boxed{\delta_2 = 2 \text{ mm}}$$

[Q.] [2004] The velocity profile in laminar boundary layer ... is given by ... over a flat plate is given by $\frac{V}{V_0} = \frac{3}{2}\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^3$

$\delta_x = \frac{4.795x}{\sqrt{Re_x}}$ for a free stream velocity of 2 m/s

$\rho_{air} = 1.2\ kg/m^3$; $\mu_{air} = 1.5\times10^{-5}\ kg/ms$; wall shear stress at a dist^n of 1 m from upstream end. $x = 1$

$\rightarrow \frac{\partial(V/V_0)}{\partial(y/\delta)} = \frac{3}{2}(1) - \frac{1}{2}\,3\left(\frac{y}{\delta}\right)^2$

$\left.\frac{\partial(V/V_0)}{\partial(y/\delta)}\right|_{y=0,\,x=1} = \frac{3}{2\delta}$

$\tau_0 = \mu\frac{\partial u}{\partial y} = \mu\frac{3}{2}V_0$

$\tau_{y=0}\Big]_{x=1} = \mu\left[\frac{du}{dy}\right]_{y=0;\,x=1}$

$= \mu\frac{d}{dy}\left[V_0\left\{\frac{3}{2}\frac{(1)}{\delta} - \frac{1}{2}\frac{3y^2}{\delta^3}\right\}\right]_{y=0}$

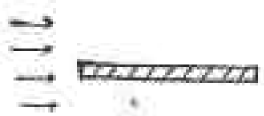
$\tau_0 = \frac{3\mu V_0}{2\delta}$

So, $\tau_0 = \frac{3\mu V_0}{2\left(\frac{4.795x}{\sqrt{Re_x}}\right)}$

$= \frac{3\times1.5\times10^{-5}\times2}{2\times4.795\times1} \sqrt{\frac{1.2\times2\times1}{1.5\times10^{-5}}}$

$\boxed{\tau_0 = 3.75\times10^{-3}\ N/m^2}$

[ES] $\frac{V}{V_0} = C_0 + C_1\frac{y}{\delta} + C_2\left(\frac{y}{\delta}\right)^2 + C_3\left(\frac{y}{\delta}\right)^3$



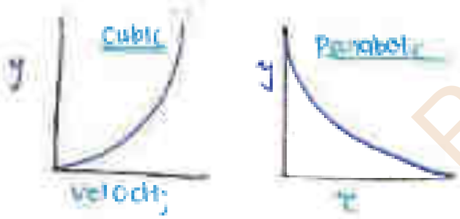
velocity = (δ)
$\tau = (\delta)$
$\delta = f(x, Re)$

(i) $y = 0$; $V = 0$

(ii) $y = \delta$; $\left(\frac{du}{dy}\right) = 0$

(iii) $y = \delta$; $V = V_0$

(iv) $\left[\frac{d^2u}{dy^2}\right] = 0$; $y = 0$



$\rightarrow \frac{V}{V_0} = \frac{3}{2}\frac{y}{\delta} - \frac{3}{2}\left(\frac{y}{\delta}\right)^3 \leftarrow$ Cubic

$\rightarrow \tau = \mu\left(\frac{du}{dy}\right) = \mu V_0\left[\frac{3}{2\delta} - \frac{3}{2}\frac{y^2}{\delta^3}\right] \leftarrow$ parabolic

$\theta = \int_0^\delta \frac{V}{V_0}\left[1 - \frac{V}{V_0}\right]dy$; $\frac{\tau_0}{\rho V_0^2} = \frac{d\theta}{dx}$

[illegible] Air flows past a [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] [illegible] the
-09 flow becomes turbulent at $Re = 2\times10^5$; $\rho_{air} = 1.2$ kg/m³
$\nu_{air} = 1.5\times10^{-5}$; find the velocity at which flow becomes turbulent

→ $Re = \dfrac{\rho V D}{\mu}$

$V = \dfrac{1.5\times10^{-5}\times2\times10^5}{40\times2}$

$Re_{local} < R_{critical}$

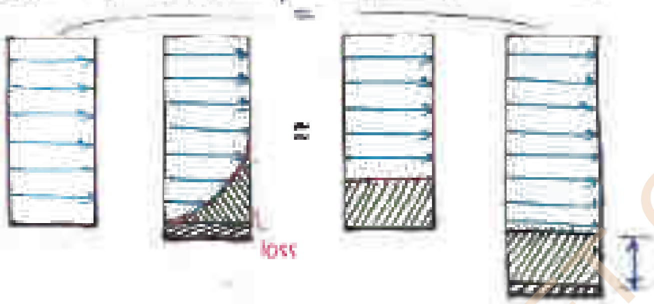
$\dfrac{\rho V D}{\mu} < 2\times10^5$

$\dfrac{VD}{\nu} < 2\times10^5 \Rightarrow V = \dfrac{2\times10^5\times1.5\times10^{-5}}{40\times10^{-3}}$

$\boxed{V = 0.075 \text{ m/s}} \Rightarrow \boxed{V = 75 \text{ m/s}}$

* Displacement thickness :- δ^*

— It is a distance by which the boundary has to be shifted in order to compensate to loss in flow rate in account of boundary layer formation.



$AV = Q$
mv = momentum
$\frac{1}{2}mv^2$ = energy

✓ **Displacement thickness :** δ^*

loss in flow rate → $$\delta^* = \int_0^\delta \left[1 - \frac{V}{V_0}\right] dy$$

✓ **Momentum thickness :-** θ

loss in momentum → $$\theta = \int_0^\delta \frac{V}{V_0}\left[1 - \frac{V}{V_0}\right] dy$$

✓ **Energy thickness :-** δ^E

loss in energy → $$\delta^E = \int_0^\delta \frac{V}{V_0}\left[1 - \left(\frac{V}{V_0}\right)^2\right] dy$$

$\delta^* > \delta^E > \theta$

$$\boxed{\text{Shape factor} = \frac{\delta^*}{\theta}}$$

Eg: $\delta^* : \delta^E : \theta$

$\rightarrow \delta^* = \int_0^\delta \left[1 - \frac{V}{V_0}\right] dy$

$= \int_0^\delta \left[1 - \frac{y}{\delta}\right] dy = \left[y - \frac{y^2}{2\delta}\right]_0^\delta = \frac{\delta}{2}$

$\rightarrow \delta^E = \int_0^\delta \frac{y}{\delta}\left[1 - \left(\frac{y}{\delta}\right)^2\right] dy = \left[\frac{y^2}{2\delta} - \frac{1}{\delta^3}\frac{y^4}{4}\right]_0^\delta$

$= \frac{\delta}{2} - \frac{1}{\delta^3}\cdot\frac{\delta^4}{4} = \frac{\delta}{2} - \frac{\delta}{4} = \frac{\delta}{4}$

$\rightarrow \theta = \int_0^\delta \frac{y}{\delta}\left[1 - \left(\frac{y}{\delta}\right)\right] dy = \frac{y^2}{2\delta} - \frac{1}{\delta^2}\frac{y^3}{3} = \frac{\delta}{2} - \frac{\delta}{3} = \frac{\delta}{6}$

$\delta^* : \delta^E : \theta = \frac{\delta}{2} : \frac{\delta}{4} : \frac{\delta}{6} \times 12$

$$\boxed{\delta^* : \delta^E : \theta = 6 : 3 : 2}$$

Note:

For ideal plate laminar fl

$$\boxed{\frac{V}{V_0} = \frac{y}{\delta}}$$

Note:

① Laminar $\boxed{\frac{V}{V_0} = \frac{y}{\delta}}$

$\delta^* = \frac{\delta}{2}$; $\delta^E = \frac{\delta}{4}$; $\theta = \frac{\delta}{6}$

② Turbulent $\boxed{\frac{V}{V_0} = \left(\frac{y}{\delta}\right)^{1/7} = \left(\frac{y}{\delta}\right)^{1/m}}$

$\delta^* = \frac{\delta}{8}$; $\boxed{\delta^* = \frac{\delta}{m+1}}$

③ Shear stress $\boxed{\tau = \tau_0\left[1 - \frac{y}{\delta}\right]}$

$\boxed{\delta^* = \frac{\delta}{3}}$; $\boxed{\theta = \frac{2}{15}\delta}$

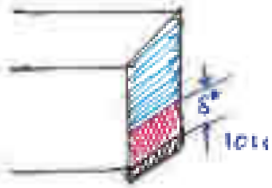
A flow over a ... power law $\frac{\delta^*}{\delta} = ?$

— 1/6 th power law : $\frac{v}{V_0} = \left(\frac{y}{\delta}\right)^{1/6}$; $m = 6$

$$\delta^* = \frac{\delta}{m+1} \Rightarrow \boxed{\frac{\delta^*}{\delta} = \frac{1}{6}} \text{ Ans}$$

A5) For a flow of an air stream with freestream velocity $V_0 = 10$ m/s, $\rho_{air} = 1.2$ kg/m³ ; $\nu = 1.5 \times 10^{-5}$ at an instant the displacement thickness $\delta^* = 0.5$ mm where boundary layer thickness is 2 mm calculate loss in flow rate on account of boundary layer formation in kg per metre width per sec (?)

$$Re = \frac{1.2 \times 10 \times 2}{1.5 \times 10^{-5}} = 5 \times 10^5$$



$$Q_{loss} = A_{loss} \times V_0$$
$$= \delta^* \times \text{width} \times V_0$$
$$= 0.5 \times 10^{-3} \times 1 \times 10$$

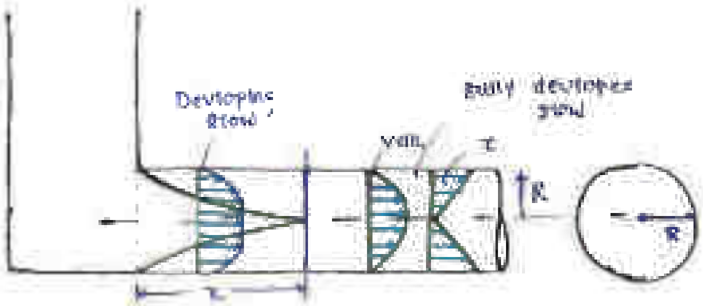
$\boxed{Q_{loss} = 5 \times 10^{-3}}$ ~~kg/m/sec~~ m³/sec

but answer is asked in kg/s (per metre width)

$$\dot{m}_{loss} = \rho Q_{loss} = 1.2 \times 5 \times 10^{-3}$$
$$= 6 \times 10^{-3} \text{ kg/s} \quad \text{Ans}$$

→ Laminar flow :

1) Through circular pipe

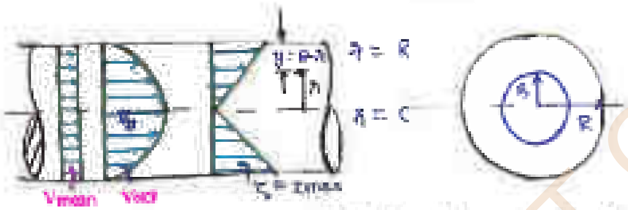


$\boxed{\delta_{max} \text{ (possible)} = R}$

Developing flow @ entrance length 'x'

$\frac{x}{D} = 0.07 Re$	(Laminar)
$\frac{x}{D} = 50$	(Turbulent)

→ shear principal → $\boxed{\frac{\partial \tau}{\partial y} = \frac{\partial P}{\partial y}}$

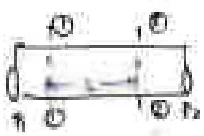


$r = 0$, at centre , $V = V_0 = V_{max}$; $\tau = \tau_{min} = 0$

$r = R$, at wall , $V = 0$, $\tau = \tau_0 = \tau_{max}$

$\boxed{\tau = -\mu \left(\frac{dy}{dr}\right)}$ Here $y = R - r$ → $dy = -dr$

Radius r → $\boxed{V = \left(-\frac{\partial P}{\partial x}\right)\left(\frac{R^2 - r^2}{4\mu}\right) = \frac{(P_1 - P_2)(R^2 - r^2)}{4\mu L}}$

→ 

$\frac{\partial P}{\partial x} = \frac{P_2 - P_1}{x_2 - x_1} = \left(\frac{P_2 - P_1}{L}\right)$

∂P

→ $V_0 = V_{max} = V_{r=0} = \left(-\frac{\partial P}{\partial x}\right)\left(\frac{R^2 - 0}{4\mu}\right)$

$\boxed{V_0 = \left(-\frac{\partial P}{\partial x}\right)\left(\frac{R^2}{4\mu}\right)}$

$$\frac{V}{V_0} = \frac{\left(-\frac{\partial p}{\partial x}\right)\frac{R^2-r^2}{4\mu}}{\left(-\frac{\partial p}{\partial x}\right)\left(\frac{R^2}{4\mu}\right)} = \frac{R^2-r^2}{R^2}$$

$$\boxed{\frac{V}{V_0} = 1 - \left(\frac{r}{R}\right)^2}$$

⟹ <u>mean velocity</u> :

$$\boxed{V_{mean} = \frac{V_c}{2}} \rightarrow V_{mean} = \left(-\frac{\partial p}{\partial x}\right)\frac{R^2}{8\mu}$$

— $Q = A V_{mean} \rightarrow V_{mean} = \frac{Q}{\pi R^2}$

$$Q = \int v\, dA$$

$$\boxed{V_{mean} = \frac{1}{A}\int V\, dA}$$

➡ <u>Hagen - Poiseuille's Equation</u> :

$$Q = \int dQ = \int v\, dA$$

$$= \int_{r=0}^{R} \left(-\frac{\partial p}{\partial x}\right)\left(\frac{R^2-r^2}{4\mu}\right) 2\pi r\, dr$$

$$Q = \left(-\frac{\partial p}{\partial x}\right)\frac{2\pi}{4\mu}\int_0^R (R^2 r - r^3)\, dr$$

$$Q = \left(-\frac{\partial p}{\partial x}\right)\frac{2\pi}{4\mu}\left[R^2\frac{r^2}{2} - \frac{r^4}{4}\right]_0^R$$

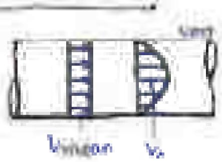
$$\boxed{Q = \left(-\frac{\partial p}{\partial x}\right)\frac{\pi R^4}{8\mu} = \frac{(P_1-P_2)\,\pi D^4}{128\mu L}}$$

→ $V_{mean} = \frac{Q}{\pi R^2}$

$$= \left(-\frac{\partial p}{\partial x}\right)\frac{R^2}{8\mu}$$

$$\boxed{V_{mean} = \frac{V_c}{2}}$$

Shortcut



→ Revolution of (rectangle) of Vmean gives cylinder with Vmean thickness

→ Revolution of parabola gives paraboloid with Vmax thickness.

$\pi R \, V_{mean} = \frac{1}{2} \pi R \, V_0$

$$\boxed{V_{mean} = \frac{V_0}{2}}$$

NOTE $\frac{V}{V_0} = 1 - \left(\frac{r}{R}\right)^m \Rightarrow V_{mean} = \left(\frac{m}{m+2}\right) V_{max}$

→ $V_{mean} = \left(\frac{P_1 - P_2}{L}\right)\left(\frac{(D/2)^2}{8\mu}\right)$

$$\boxed{P_1 - P_2 = \frac{32 \mu V L}{D^2}} \quad (\text{mean velocity}) \qquad \left(\frac{P_1 - P_2}{L}\right) \propto V$$

$E_1 = E_2 + h_{loss}$

$\left[z_1 + \frac{P_1}{\rho g} + \frac{V_1^2}{2g}\right] = \left[z_2 + \frac{P_2}{\rho g} + \frac{V_2^2}{2g}\right] + h_f$

$h_f = \frac{P_1 - P_2}{\rho g}$

same as $h_f = \frac{fLV^2}{2gd}$

for laminar $f = \frac{64}{Re} \propto \frac{1}{V}$

$$\boxed{h_f \propto V}$$

NOTE → pressure drop $(P_1 - P_2)$ in circular pipe is proportional to velocity

$$\boxed{\frac{P_1 - P_2}{L} \propto V}$$

when velocity across is const for given m then (Head loss / L) is proportional to V^2

$$\boxed{\left(\frac{P_1 - P_2}{L}\right) \propto h_f \propto V^2} \quad [\text{valid when } f \text{ is const}]$$

⇒ Wall shear stress :-

$\tau = \tau_{0, r=R} = -\mu\left(\frac{\partial u}{\partial r}\right) = -\mu \frac{\partial}{\partial r}\left[\left(\frac{-\partial p}{\partial x}\right)\left(\frac{R^2 - r^2}{4\mu}\right)\right]_{r=R}$

$= -\mu\left(\frac{-\partial p}{\partial x}\right)\frac{1}{4\mu}\left[0 - 2R\right]$

$$\boxed{\tau_0 = \left(\frac{-\partial p}{\partial x}\right)\frac{R}{2}}$$

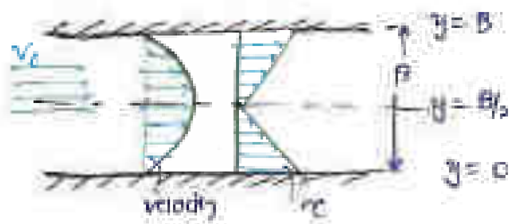
NOTE → $V_{mean} = V_{act}$ @ $0.707R$ from centre

→ $\left(\frac{-\partial p}{\partial x}\right)\frac{R^2}{8\mu} = \left(\frac{-\partial p}{\partial x}\right)\left(\frac{R^2 - r^2}{4\mu}\right)$

$r^2 = R^2 - R^2/2$ or $r = R/\sqrt{2}$

$$\boxed{r = 0.707R}$$

2) Between parallel plates [both are fixed]



@ $y = 0, B$

$V = 0$, $\tau = \tau_0 = \tau_{max}$

@ $y = B/2$

$V = V_{max}$, $\tau = 0 = \tau_{min}$

$$\boxed{v = \left(\frac{-\partial p}{\partial x}\right)\left(\frac{By - y^2}{2\mu}\right)}$$

→ $\underline{V_{max}}$: $\boxed{V_{max} = V_{y=B/2} = \left(\frac{-\partial p}{\partial x}\right)\frac{B^2}{8\mu}}$

→ $\underline{V_{mean}}$ ⇒ $V_{mean} = \frac{1}{A}\int v\, dA$ ⇒ $V_{mean} = \left(\frac{-\partial p}{\partial x}\right)\frac{B^2}{12\mu}$

$\boxed{V_{mean} = \frac{2}{3} V_c}$ (for fixed parallel plates)

→ $P_1 - P_2$ ⇒

$$V_{mean} = \left(\frac{-\partial p}{\partial x}\right)\frac{B^2}{12\mu} = \frac{P_1 - P_2}{L}\,\frac{B^2}{12\mu}$$

$$\boxed{P_1 - P_2 = \frac{12\mu V L}{B^2}}$$

→ Head loss (h_f)

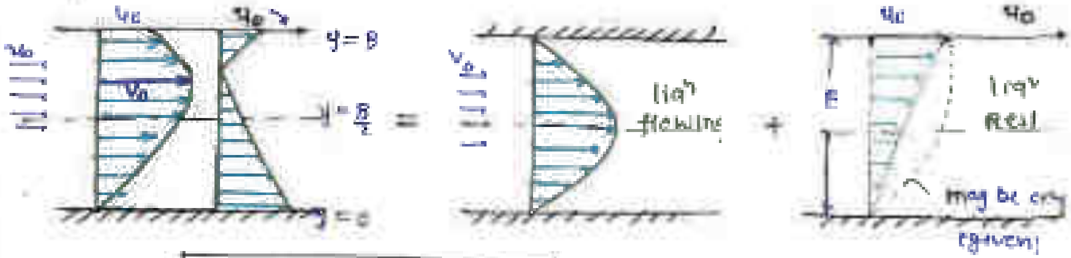
$$\boxed{h_f = \frac{P_1 - P_2}{\rho g} = \frac{12\mu V L}{\rho g B^2}}$$

→ Wall shear stress

$$\tau_0 = \tau_{y=0,B} = \mu\left(\frac{du}{dy}\right)_{y=0}$$

$$\boxed{\tau_0 = \left(\frac{-\partial p}{\partial x}\right)\frac{B}{2}}$$

8) Couette flow
Laminar flow fixed & moving parallel plates
(Special case)



$$\boxed{u = \left(-\frac{\partial P}{\partial x}\right)\frac{(By - y^2)}{2\mu} + \frac{u_0 y}{B}}$$

— formulas:

(1) circular pipe

1) Boundary cond^n

$r = 0$; $V = V_0$, $\tau = 0$

$r = R$; $V = 0$, $\tau = \tau_0$

$\rightarrow V = \left(-\frac{\partial P}{\partial x}\right)\left(\frac{R^2 - r^2}{4\mu}\right)$

2) $V_{mean} = \frac{V_{max}}{2}$ & $V_{mean} = \left(\frac{m}{m+2}\right) V_{max}$

$= 1 - \left(\frac{r}{R}\right)^m$

3) $\frac{V}{V_0} = 1 - \left(\frac{r}{R}\right)^2$

4) $Q = \left(-\frac{\partial P}{\partial x}\right)\frac{\pi R^4}{8\mu} = \frac{(P_1 - P_2)\pi D^4}{128\,\mu L}$

5) $P_1 - P_2 = \frac{32\mu V L}{D^2}$

6) $\tau_0 = \left(-\frac{\partial P}{\partial x}\right) R/2$

(2) parallel pipes

1) $V_{mean} = \frac{2}{3} V_{max}$

i) $V = \left(-\frac{\partial P}{\partial x}\right)\left(\frac{By - y^2}{2\mu}\right)$

2) $P_1 - P_2 = \frac{12\mu V L}{B^2}$; $h_f = \frac{P_1 - P_2}{\rho g}$

3) $\tau_0 = \left(-\frac{\partial P}{\partial x}\right)\frac{B}{2}$

GATE -11 The max. velocity in a one dimensional fully developed flow between two fixed parallel plates is 6 m/s then Vmean = (?)

$\rightarrow$ $V_{mean} = \frac{2}{3} V_{max}$ (fixed plate)

$= \frac{2}{3}(6) = 4$ m/s

GATE -2015 Laminar through flow circular pipe of radius 'R' the local velocity at any radius r is given by

$V = \frac{R^2}{4\mu}\left[\frac{-\partial P}{\partial x}\right]\left[1 - \frac{r^2}{R^2}\right]$ then the mean velocity of flow = ?

$V_{mean} = \frac{1}{2} V_{max}$

V_{max} @ $r = 0$ $\Rightarrow$ $V_{max} = \frac{R^2}{4\mu}\left(\frac{-\partial P}{\partial x}\right)$

$V_{mean} = \frac{R^2}{8\mu}\left(\frac{-\partial P}{\partial x}\right)$

GATE -16 The pressure drop for relatively low Re No flow for circular pipe of 600 mm dia is 70 kPa over length of 30 m then wall shear stress = ?

$\tau_0 = \left(\frac{-\partial P}{\partial x}\right)\frac{R}{2}$

$= \left(\frac{P_1 - P_2}{L}\right)\frac{R}{2} = \left(\frac{70\times10^3}{30}\right)\left(\frac{0.3}{2}\right)$

$\boxed{\tau_0 = 350\ N/m^2}$

GATE 06 In laminar flow a circular pipe of diameter D & the local velocity at radius r, Velo. $u = u_0\left[1 - \frac{4r^2}{D^2}\right]$ then pressure drop across length 'L' of pipe is

$\rightarrow$ $V = V_0\left[1 - \frac{4r^2}{D^2}\right] = V_0\left[1 - \frac{r^2}{R^2}\right]$

From std. eq^n (above)

$P_1 - P_2 = \frac{32\mu VL}{D^2}$ It is Vmean

$V_{mean} = \frac{V_0}{2}$

$\boxed{P_1 - P_2 = \frac{16\mu V_0 L}{D^2}}$

ESE) The [illegible] value of [illegible]
laminar flow through circular pipe ?
(a) 0.005 (b) 0.02 (c) 0.032 (d) 0.04

For circular pipe;

$$f = \frac{64}{Re},\ f_{min} = \frac{64}{2000} = 0.032$$

→ for open: $f = 0.005$ to 0.01
$f = 0.02$ to 0.04

ts 6.15) In laminar flow through circular pipe of radius 10 cm with mean velocity of 5 m/s the local velocity at radial dist^n 5 cm

→ $V_{mean} = 5\ m/s = V$
$R = 10$ cm
$r = 5$ cm

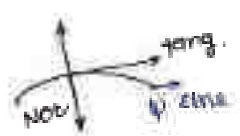
$V_{mean} = \frac{V_0}{2} \Rightarrow \boxed{V_0 = 10\ m/s}$

$$\frac{V}{V_0} = 1 - \left(\frac{r}{R}\right)^2 = 1 - \left(\frac{5}{10}\right)^2 = 0.75$$

$\boxed{V = 7.5\ m/s}$ local velocity.

⇒ Fluid Kinematics (V-1 Problem)

CRO [8]



$V = f[s, n, t]$

$V < \begin{matrix} s \to V_s \to a_s \\ n \to V_n \to a_n \end{matrix}$

$$a_s = V_s \frac{\partial V_s}{\partial s} + V_n \frac{\partial V_s}{\partial n} + \underbrace{\left(\frac{\partial V_s}{\partial t}\right)}_{\text{steady} = 0}$$

$$a_n = V_s \frac{\partial V_n}{\partial s} + V_n \frac{\partial V_n}{\partial n} + \left(\frac{\partial V_n}{\partial t}\right)$$

$$\boxed{a_s = V_s \frac{\partial V_s}{\partial s}}$$

$$\boxed{a_n = \frac{V_s^2}{R}}$$

$R = 9\ m$, $\frac{\partial V_s}{\partial s} = \frac{1}{3}\ m/s/m$

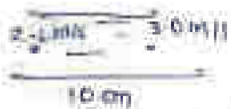
$a_t = \sqrt{a_s^2 + a_n^2}$

$a_s = V_s \frac{\partial V_s}{\partial n} = 3 \cdot \frac{1}{3} = 1$

$a_n = V_s^2/R = 3^2/9 = 1$

$\rightarrow a_t = \sqrt{a_s^2 + a_n^2} = \sqrt{2}$

CRO $\boxed{9}$



$a_s = V_s \frac{\partial V_s}{\partial s}$ — no given

$= \frac{2.5 + 3.0}{2} \cdot \frac{[3.0 - 2.5]}{10}$

$=$ ____ m/s^2

CRO $\boxed{10}$

$R = 30\ m$

$a_{s\,(\theta = 45^\circ)}$

$ds = R\,d\theta$

$V_s = 3\sin\theta$

$a_s = V_s \frac{\partial V_s}{\partial s}$

$= 3\sin\theta \frac{\partial}{\partial s}[3\sin\theta]$

$= 3\sin\theta \frac{\partial}{R\,d\theta}[3\sin\theta]$

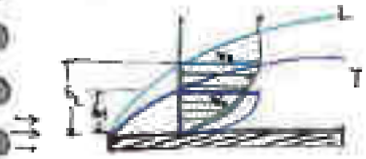
$= 3\sin\theta \frac{1}{R}(3)\cos\theta = 3\sin 45^\circ \cdot \frac{1}{3}(3)\sin 45^\circ$

$\boxed{a = 1.5\ m/s^2}$

→ For circular pipe :

$V_{mean} = 0.82 \text{ to } 0.85\, V_{max}$

Note: The growth of boundary layer will be faster in turbulent zone compare to laminar zone.



✓ $Loss_L > Loss_T$

✓ $V_{mean_L} < V_{mean_T}$

- in lar

Reason: in laminar zone all layer are same so to reach max level time req^n is more.

$$\boxed{\tau = \underbrace{\mu\left(\frac{du}{dy}\right)}_{\text{viscous shear}} + \underbrace{\eta\left(\frac{du}{dy}\right)}_{\text{Eddy shear}}}$$

$$= \mu\left(\frac{du}{dy}\right) + \rho l^2\left(\frac{du}{dy}\right)^2$$

where; η = ~~fluid property~~ eddy shear co-efficient
l = Prandtl's mixing length
μ = fluid property
η = flow property
[High Re, $\eta >> \mu$] $\boxed{\eta >>> \mu}$ High Re

⇒ Hydrodynamically (Smooth & Rough) ⟶ $\delta' = \frac{11.6\nu}{V^*}$

$Re_1 < Re_2$

$\delta_1' > \delta_2'$

$K = K_s$ = Avg surface roughness size

strike the boundary then it is considered as Hydrodynamically Rough otherwise (laminar sub layer covering all the surface (irregularity) hydrodynamic smooth.

- For a Highly turbulent flow (for High Re) the flow may behave like as Hydrodynamically rough.

⇒ Re ↑

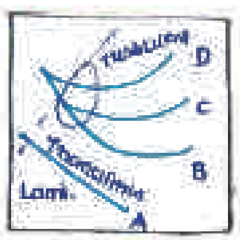
Laminar
Transitional
Turbulent

Hyd. smooth dyn
Transi.
Hyd. rough

⇒ <u>Moody's chart</u> / <u>Diagram</u>

Relates to [f, Re]

<u>GATE</u> :

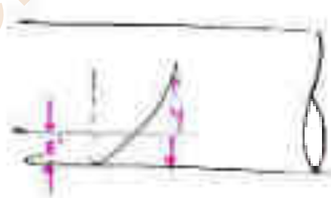


Moody's chart's depends [f, Re]

A → Laminar
B → smooth hyd. dynamic
D → Rough hyd. dyn

⇒ $V = f\left(-\frac{\partial P}{\partial x}\right)$

<u>Hydrodynamic smooth</u>



$0 < y < \delta' \rightarrow$ linear

$y > \delta' \rightarrow$ logarithmic

$$\frac{V}{V^*} = 5.75 \log_{10}\left(\frac{V^* y}{\nu}\right) + 5.5$$

shear velocity

Kinematic viscosity

<u>Hydrodynamic Rough</u>

$y > 0 \rightarrow$ logarithmic

$$\frac{V}{V^*} = 5.75 \log_{10}\left(\frac{R}{K_s}\right) + 8.5$$

surface roughness

→ [illegible]

(1) $V_{mean} = 0.82$ to $0.85 \, V_{max}$

(2) $\dfrac{V_{max} - \bar{V}}{V^*} = 3.75$

where $\bar{V}$ = mean velocity
V^* = shear velocity

(3) $\dfrac{1}{\sqrt{F}} = 2.0 \log (R/k) + 1.74$ (Hyd. rough)

(4) $V^* = \sqrt{\dfrac{\tau_0}{\rho}} = \sqrt{\dfrac{\rho F V^2}{8 \rho}} = \sqrt{\dfrac{F}{8}} \, V$ — mean velocity

(5) $\tau_0 = \dfrac{\rho F V^2}{8}$ — mean velocity

(6) $V_* = \sqrt{\dfrac{F}{8}} \cdot V$ or $V_{max} = (1 + 1.33\sqrt{F}) V_{mean}$

Ex-09) Water flows through rough pipe of 100 mm dia at 50 L/s the avg. roughness surface size $k_s = 0.15$ mm. Find max velocity, mean velocity, friction factor, shear velocity

1) → $V_{mean} = \dfrac{Q}{A}$ ∴ $Q = 50 \times 10^{-3} \, m^3/s$
$A = \pi/4 \, (0.1)^2 = 0.00785$

$= \dfrac{50 \times 10^{-3}}{0.00785}$ | from given V_{mean} find

$\boxed{V_{mean} = 6.369 \, m/s}$

2) → $V_{mean} = (0.82 \text{ to } 0.85) V_{max}$
$6.36 = 0.85 \, V_{max}$

Here k_s = surface roughness given so go through total process first find F

$\dfrac{1}{\sqrt{F}} = 2.0 \log_{10} \left(\dfrac{R}{k_s} \right) + 1.74$

$= 2.0 \log_{10} \left(\dfrac{50}{0.15} \right) + 1.74$

$\dfrac{1}{\sqrt{F}} = 6.7457$

$\boxed{F = 0.0217}$

$\Rightarrow \quad V^* = \sqrt{\frac{f}{8}}\,(\bar{V})$

$= \sqrt{\frac{0.021}{8}} \times 6.36$

$\boxed{V^* = 0.33 \text{ m/s}}$ ← shear velocity

b) $\frac{V_{max} - \bar{V}}{V^*} = 3.75$

$V_{max} - 6.35 = (3.75)(0.33)$

$\boxed{V_{max} = 7.55 \text{ m/s}}$

c) $\tau_0 = \frac{\rho f \bar{V}^2}{8} = \frac{1000 \times 0.021 \times (6.36)^2}{8}$

$\boxed{\tau_0 = 106 \text{ N/m}^2}$

⇒ Archemedis principle :-

- Whenever an object is emmersed (submerged) either completely or partially it will be lifted up by (F_B) Buoyant force whose magnitude is equal to weight of fluid displaced. F_B always act vertically upward, through center of Buoyancy (COB), COB is the center of gravity for displaced volume.

Bouyance force (F_B)

$$\boxed{F_B = \text{weight of fluid displaced}}$$

$W_{Body} = F_B$ [freely floating body]

$T + W = F_B$

(8) Metacentre - 'M'

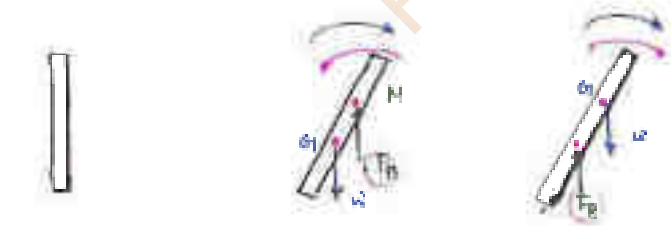
Equilibrium condⁿ

① Partially submerged [M & G]

1) stable equilibrium ⟶ M lies above G, $\overline{GM} > 0$

2) unstable equilibrium ⟶ M lies below G, $\overline{GM} < 0$

3) Neutral equilibrium ⟶ M coincide with G, $\overline{GM} = 0$



Neutral   Stable equilib   unstable e..

1) stable equilibrium → B above G
2) unstable equil^n → B below G
3) Neutral equil^n → B on G

NOTE

For better stability : High metacentric height desirable
but for comfort cond^n → low metacentric height is req^r

→ Time period of oscillation

$$\boxed{t = 2\pi\sqrt{\frac{K_G^2}{g(GM)}}}$$

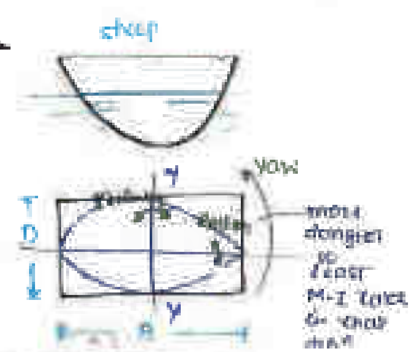
where K_G = Radius of gyration
$I = mK_G^2$

$$\boxed{t \propto \frac{1}{\sqrt{GM}}}$$

Metacentric Height (GM)

$$\boxed{\begin{array}{l} GM = \overline{BM} - \overline{BG} \\ \overline{GM} = \frac{I}{V} - \overline{BG} \end{array}}$$

where I = least moment of area at water surface level
V = Immersed vol^n



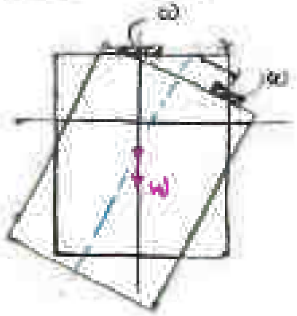
$b > d$

$I_{xx} < I_{yy}$

$\frac{bd^3}{12} < \frac{db^3}{12}$

safe

→ Experimental method to find metacentric height (GM)

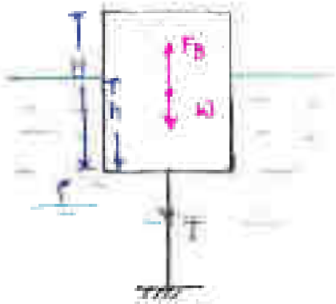


$$\boxed{\overline{GM} = \frac{w \cdot x}{(W + w)\tan\theta}}$$

$$\boxed{\overline{GM} = \frac{w \cdot x}{W \tan\theta}}$$

→ F_B = wt of fluid displaced

Ex A cylinder body, Area A, density Height H, density ρ_c was immersed in liq^n of density ρ, where is depth of h time to bottom within string tension in the spring will be T = (?)



$T + W = F_B$

$= F_B - W$

$=$ wt of fluid disp − wt of body

$= V_{liq} \times Vol_{liq} - V_{bod} \times Vol_{body}$

$= \rho g A h - \rho_c g A H$

$$\boxed{T = [\rho h - \rho_c H]\, g A}$$

Ex An object P was floating on a water with half of its volume inside and object Q with $2/3^{rd}$ of vol^n inside the water then the ratio of sp. gr. Q to sp. gr. P

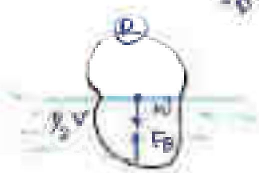
$\frac{S_Q}{S_P} = (?)$

a) $1/2$
b) $1/3$
c) $3/4$
d) $4/3$

Here Q have $2/3^{rd}$ vol^n & P have $1/2$ vol^n

S_Q Q (heavier) > P (less wt)

$S_Q > S_P \Rightarrow S_Q/S_P > 1 = 4/3$ (only d)



$W_P = F_{BP}$

$V_P V_P = V_W \times V_P/2$

$S_P = \frac{V_P}{V_W} = \frac{1}{2}$

$W_Q = F_{BQ}$

$V_Q V_Q = V_W \times 2/3\, V_Q$

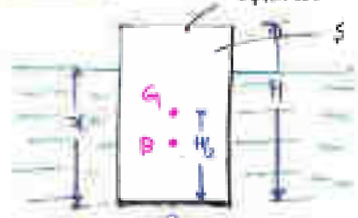
$S_Q = \frac{V_Q}{V_W} = \frac{2}{3}$

$\frac{S_p}{S_q} = \frac{1/2}{2/3} = 3/4$

$\Rightarrow \boxed{\frac{S_q}{S_p} = \frac{4}{3}}$

EC-06

1)



Rectangle

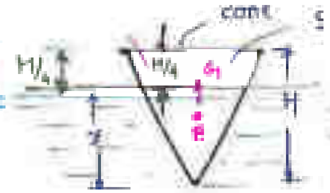
$\boxed{x = SH}$

$\overline{BG} = \overline{OG} - \overline{OB}$

$= \frac{H}{2} - \frac{SH}{2}$

$\overline{BG} = \frac{H}{2}[1 - S]$

2) (dist from base to G)



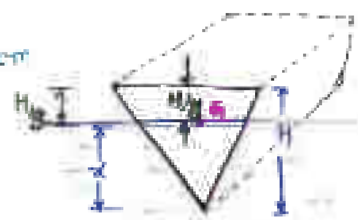
cone

$\boxed{x = S^{1/3} H}$

$\overline{BG} = \frac{3}{4}(H - x)$

$\overline{BG} = \frac{3}{4} H [1 - S^{1/3}]$

3) (dist from base to CG)



Triangular Beam

$\boxed{x = S^{1/2} H}$

$\overline{BG} = \frac{2}{3}[H - x]$

$\overline{BG} = \frac{2}{3} H [1 - S^{1/2}]$

— table

ES-09

Height $H = 2d$, (two times of dia) floating in the water with its axis vertical will be

a) SE
b) USE
c) NE
D) x

$\overline{GM} = \overline{BM} - \overline{BG} \begin{matrix} = \\ < \\ > \end{matrix} 0$

$\frac{I}{V}$

$\overline{BM} \bigcirc \overline{BG}$

$\overline{BG} = 0.4d$

$BM = \frac{I}{V}$

$= \frac{\frac{\pi}{64} d^4}{\frac{\pi d^2}{4} \times 1.2d}$

$BM = \left(\frac{4}{64 \times 1.2}\right) d = 0.058d$

$\rightarrow \left(\frac{4}{64 \times 1.2}\right) d < 0.4d$

$\boxed{GM < 0} \Rightarrow BM - BG < 0 \Rightarrow$ unstable

$\Rightarrow$ so, M is lies below's 'G'.

ES A spherical & cube having same surface area, are completely immersed in water then the ratio of loss of wt. on sphere to cube. (?)

$\rightarrow$

$A_{S_s} = A_{S_c}$

$6a^2 = 4\pi r^2$

$a = \sqrt{\frac{2\pi}{3}}\, r$

$\frac{(F_B)_{sph}}{(F_B)_{cube}} = \frac{\text{loss of wt sphere}}{\text{loss of wt cube}}$

$\frac{(F_B)_{sphere}}{(F_B)_{cube}} = \frac{\gamma_w \times Vol_s}{\gamma_w \times Vol_c}$

$= \frac{4/3 \pi r^3}{a^3} = \frac{4}{3} \frac{\pi \cdot r^3}{\left(\frac{2}{3}\pi\right)^{3/2} r^3} = \frac{4}{3} \cdot \frac{3^{3/2}}{2^{3/2}} \cdot \frac{\pi}{\pi^{3/2}}$

$= \frac{(2)^2}{(2)^{3/2}} \cdot \frac{(3)^{3/2}}{(3)} \cdot \frac{1}{\sqrt{\pi}} = \frac{\sqrt{2} \cdot \sqrt{3}}{\sqrt{\pi}} = \sqrt{\frac{6}{\pi}}$

&

$4\pi r^2 = 6s^2$

$\frac{r^2}{s^2} = \frac{6}{4\pi}$ & $\frac{r}{s} = \sqrt{\frac{6}{4\pi}}$

$\frac{(F_B)_s}{(F_B)_c} = \frac{4/3 \pi r^3}{a^3} = \frac{4}{3}\pi \left(\frac{r^2}{s^2}\right)\left(\frac{r}{s}\right) = \frac{4}{3}\pi \left(\frac{6}{4\pi}\right)\left(\sqrt{\frac{6}{4\pi}}\right)$

$= \sqrt{6/4\pi}$

HPSC (ME) ES-AE A object weighing 100 N in air was found to weigh 80 N in water then relative density of object will be

$S_0 = (?)$

$(F_B)_{air} = 100\ N$

$(F_B)_w = 80\ N$

Relative density of object

$$\frac{W_{air}}{W_{air} - W_{water}} = R.D\ \text{object}$$

$$\frac{100}{100-80} = 5 = S_0$$

- Because of Buoyance force:
This is measurement value of object in air & water is given.
It is not actual weight

By purchasing any thing we we found value of weight we calc: wt in air so now so both are same

$W' = W - F_B$

$100 = W - \gamma_{air} \times vol^m$

$100 = W = \gamma_b \times V$ — ①

$W' = W - F_B$

$80 = W - \gamma_w \times vol^m$

$80 = 100 - \gamma_w \times vol$

$\gamma_w \times vol^m = 20$ — ②

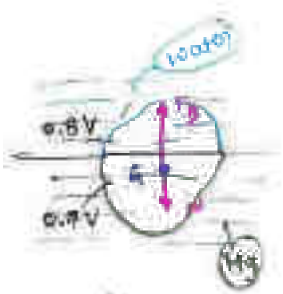
$$\frac{①}{②} = \frac{\gamma_b \times V}{\gamma_w \times V} = \frac{100}{20} = 5$$

$$\boxed{S_b = 5}$$

$$\boxed{\text{Loss of weight} = W - W' = F_B}$$

Actual wt — Apparent wt

GATE-18 An object is floating at a interface of water & mercury such that 30% of its volm inside the water then the relative density of object



$W_{body} = F_B = \text{wt of (water + Hg)}$

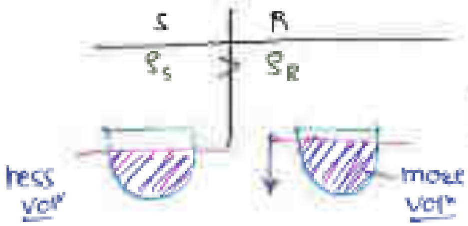
$\gamma_b \cdot V = \gamma_w \times 0.3V + \gamma_{Hg} \times 0.7V$

$\gamma_b V = \gamma_w \times 0.3V + 13.6 \times \gamma_w \times 0.7V$

$RD = \frac{\gamma_b}{\gamma_w} = 0.3 + 13.6 \times 0.7$

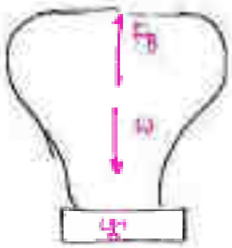
$$\boxed{RD = \frac{\gamma_b}{\gamma_w} = 9.82}$$

IAS-11) for unchanged mass ship entering from sea to water



for river:
$W = F_B = \rho_{liq} \times V_I$

IAS-11) Ballon filled with methen $\rho_{CH_4} = 0.75$ kg/m³ floating in air $\rho_{air} = 1.25$ kg/m³. the min. vol^m of balloon that can lift the man weighing 75 kg

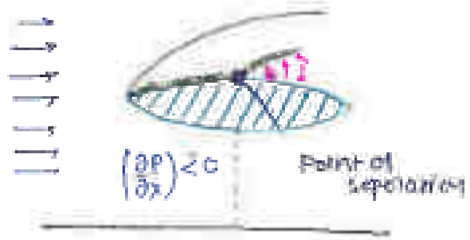


$W_{(man + CH_4)} = F_{B(air)}$

$mg + (\rho_{CH_4} g)V = (\rho_{air} g)V$

$V = \dfrac{75}{\rho_{air} - \rho_{CH_4}} = \dfrac{75}{(1.25 - 0.75)}$

$\boxed{V = 150 \text{ m}^3}$



- For a real fluid flow over a solid boundary within the boundary layer zone, for the fluid particles to move forward has to do work against friction at the expence of its kinetic energy. The loss in energy will be recovered from adjacent layer, as the work increases a stage will come at certain pt. when fluid layer may not able to stick to the boundary. Hence it leaves the boundary.
- This point is known as **point of seperation**; the seperation is desirable because behind seperation negative pressure exist.

NOTE: 1) at point of seperation

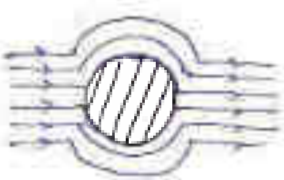
$$\left(\frac{du}{dy}\right)_{y=0} = 0$$

2) Negative pressure gradient, can prevent (Delay) seperation

$$\left[\frac{\partial P}{\partial x} < 0\right]$$

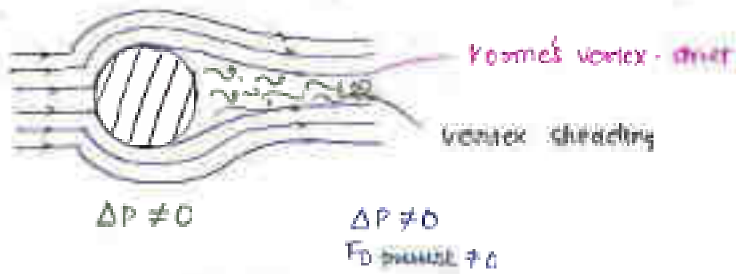
3) Any activity which can increase energy of particle near point of seperation can Delay (Prevent) the seperation.

— (Ideal) No seperation



$\Delta P = 0$

flow
① with seperation [Wide wake zone]

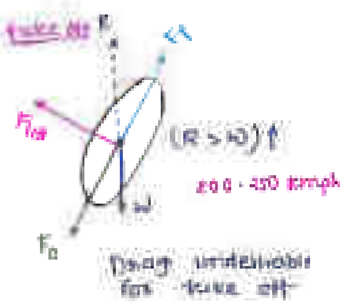


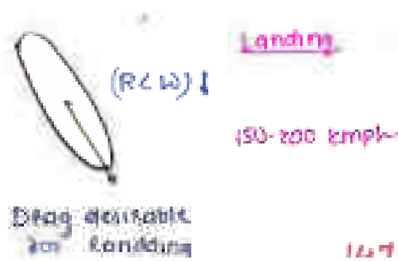
② Narrow wake zone
[even streamlined body]

⇒ Method's to prevent (delay) seperation :-

1) By injecting (or) suction fluid near point of seperation
2)
3) By streaming the body
4) By encreging turbulence

→ [Frictional Drag]





Passenger flight are subsonic zone $(M<1)$

Fighters → supersonic $(M>1)$

by moving up $\rho \downarrow$ (reduces)
(sky)

Q. Why aircraft fly above the clouds?

density of cloud more
feels huge / unbalance drag

For flight → angle of attack ↑

⇒ $\boxed{F_{drag\,(friction)} = F_D = \frac{C_D A \rho V_0^2}{2}}$

C_D = coefficient of drag

	Laminar	Turbulent
C_D avg	$\frac{1.328}{\sqrt{Re_L}}$	$\frac{0.074}{(Re_L)^{1/5}}$
C_{D_x} local	$\frac{0.664}{\sqrt{Re_x}}$	$\frac{0.058}{(Re_x)^{1/5}}$

⇒ $\boxed{F_{lift} = F_L = \frac{C_L A \rho V_0^2}{2}}$

C_L = coefficient of lift

$\boxed{C_L = 2\pi \sin\alpha}$

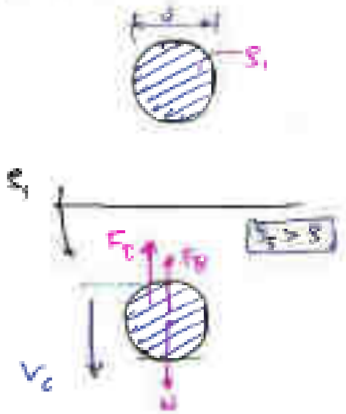
α = angle of attack
↳ $17-19°$

⇒ On sphere

$F_{D_{total}} = F_{D_{pressure}} + F_{D_{friction}}$

$\boxed{F_{D_{total}} = 3\pi\mu V d}$

→ STOKE'S EQUATION



$$W = F_B + F_{Drag}$$

$$\rho_s g \times \left[\frac{4}{3}\pi r^3\right] = \rho_l g \left[\frac{4}{3}\pi r^3\right] + 3\pi\mu V_0 d$$

$$V_0 = \frac{g \cdot \frac{4}{3}\pi \frac{d^3}{8}\left[\rho_s - \rho_l\right]}{3\pi\mu d}$$

$$\boxed{V = \frac{g d^2}{18\mu}\left[\rho_s - \rho_l\right]}$$

$$dA = B * dx$$

$$F_{drag\,(one\ side)} = \int \tau_0 * dA$$

$$F_D = \int_{x=0}^{L} \tau_0 (B\,dx)$$

Drag on one side of plate.

$$F_D = C_D\, A\, \frac{\rho V_0^2}{2}$$

→ $$C_{D_{avg}} = \frac{(F_D)}{A\left(\frac{\rho V_0^2}{2}\right)}$$

$$\frac{F_D}{A} = \tau_0 = C_D \frac{\rho V_0^2}{2} \Rightarrow C_{D_x}(\text{local}) = \frac{\tau_0}{\left[\frac{\rho V_0^2}{2}\right]}$$

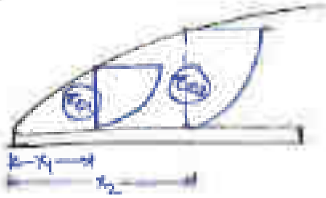
$$\frac{\tau_0}{\left[\frac{\rho V_0^2}{2}\right]} = \frac{0.664}{\sqrt{\frac{\rho V x}{\mu}}}$$

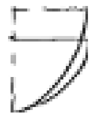
$\tau_0 \propto \frac{1}{\sqrt{x}}$ → for laminar

$\tau_0 \propto \frac{1}{(x)^{1/5}}$ → for turbulent

GATE-15
IAS

NOTE.





$\boxed{\tau_{0_1} > \tau_{0_2}}$

G-15 Flate plate of 1 m wide & 2 m long is towed through water with 2 m/s. $\nu_{water} = 10^{-6}$ m²/s. assuming boundary remain laminar total drag on both side of plate.

→ Laminar boundary

$F_D = C_D A \frac{\rho V_0^2}{2}$

$= \left[C_D A \frac{\rho V_0^2}{2} \right] 2$ (Both side)

$C_D = \frac{0.664}{\sqrt{Re_x}} = \frac{0.664}{\sqrt{\frac{2 \times 2}{10^{-6}}}} =$

→ $C_D = \frac{1.328}{\sqrt{Re_x}}$ (Laminar)

$C_D = \frac{1.328}{\sqrt{\frac{2 \times 2}{10^{-6}}}} = 6.64 \times 10^{-4}$

$F_D = 6.64 \times 10^{-4} \times (2 \times 1) \times \frac{10^3 \times 2^2}{2} \times 2$

$\boxed{F_D \simeq 5.3 \text{ N}}$

ES
G-07
NOTE

Initial zone shear zone stress is high as entrance zone

$F_{D_1} > F_{D_2}$

Shear stress ↑
Drag force ↑

$F_{D_1} > F_{D_2}$

$\frac{F_{D_1}}{F_{D_2}} = (?)$

(a) = 1
(b) < 1
(c) > 1
(d) ∞

GATE-06)
(GATE-15)

$$Q_1 = Q_2 + Q_3$$

$$Q_3 = Q_1 - Q_2$$

$$BC = ab - cd$$

$\frac{V}{V_0} = \frac{y}{\delta}$ from momentum thickness $\boxed{\delta^* = \delta/2}$

$$Q_3 = Q_1 - Q_2$$

$$\boxed{Q_3 = \delta V/2}$$

G-06) An aircraft flying in level flight at 200 kmph through air $\rho_{air} = 1.2$ kg/m³ $\mu_{air} = 1.5 \times 10^{-5}$ N·s/m²,
$C_D = 0.0085$, $C_L = 0.04$
mass of Aircraft = 800 kg
the effective length area of aircraft

→ For level flight $F_L = W$

$$F_L = W = C_L \frac{A \rho V_0^2}{2}$$

$$mg = C_L \frac{A \rho V_0^2}{2}$$

$$800 \times 9.81 = \frac{0.04 \times A \times 1.2 \times [200 \times 5/18]^2}{2}$$

$$\boxed{A = 10.6\ m^2}$$

Ex. 2009

A) → For flow over surface of cylinder the total drag will be reduced when the surface over a cylinder separates further downstream

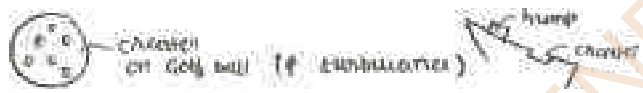
B) → When the boundary layer separates further down stream form drag will be reduce & the skin drag increases marginally.

| Form drag = press + friction drag

$\downarrow F_{total} = F_{D\,\text{pressure}} \downarrow + F_{D\,\text{skin drag}} \uparrow$

Golf the reduce drag (aim)
→ by delay separation reduce drag

to prevent



Bernoulli

FMM

(1) Bernoulli's → energy
(2) Dimensional
(3) turbomachinery (velocity triangle
(4) Fluid kinematics → or mass im
TES mainly